METHOD FOR CALCULATING SPACECRAFT LONGITUDINAL MOTION PARAMETERS DURING LANDING ON THE SURFACE OF A SMALL CELESTIAL BODY

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The article describes a method for calculating longitudinal motion parameters of a spacecraft during landing on the surface of a small celestial body. The method takes into consideration the reaction applied to the movable legs from both the surface soil and operation of the thrusters. The article discusses application of the method for dynamic analysis of the spacecraft motion during its landing on the surface of the Moon, taken as an example. In case of the vertical landing, the optimal value of the force in shock absorbers is selected, which makes it possible to obtain reasonable angles of struts after the landing with moderate forces of soil response on the legs. The authors estimate the impact of the angular orientation of the spacecraft relative to the Moon's surface on the change of the legs position, as well as the impact of soil response normal forces on the legs, and shock absorbers deformation during landing. The article contains graphs of the spacecraft longitudinal and vertical velocities, which are calculated considering the influence of legs motion during the non-simultaneous contact with the soil.

Keywords: landing on small celestial bodies, method for dynamics analysis of spacecraft motion, evaluation of influence of disturbing factors, conditions for safe landing.

The program of research into different celestial bodies of the solar system implies spacecraft flights to these bodies in the coming decades. In some cases, gathering information about a celestial body implies landing of a spacecraft on the surface of the body.

Small celestial bodies without atmosphere and with a sufficient gravitational field are of particular interest. The bodies of this type are asteroids such as Ceres, Pallas, and other satellites of the planets (the Moon near the Earth, Io and Europa near Jupiter, etc.).

At the conceptual design stage of a spacecraft, designed to landing on the surface of a small celestial body, there is a need in a method for calculating spacecraft motion parameters and estimating loads on the landing gear. The method would allow determining the main characteristics of the landing gear and parameters of the shock absorbers, as well as estimating the conditions of a safe landing on the surface of a small celestial body.

The previous publications [1–6] showed that during landing on the surface of the Moon, the most dangerous case is a longitudinal motion of the spacecraft with a lateral velocity component when one landing gear leg is touching an ascending slope of the surface. Therefore, the method for calculating landing parameters during the longitudinal motion of the spacecraft can be used at the early stages of the project definition phase.

This makes it possible to estimate the full loads on the spacecraft landing gear.

Let us formulate a mathematical model of the spacecraft motion at the last stage of landing on a small celestial body.

The main assumptions are the following ones:

a) we consider a longitudinal motion of a spacecraft with three legs;

b) aerodynamic forces are neglected;

c) gravitational acceleration is constant;

d) friction in the joints of the legs is neglected;

e) a spacecraft landing site on a celestial body is considered to be rigid.

During the development of a mathematical model of the spacecraft motion, the following coordinate systems are used:

1) the surface coordinate system (SCS) X, Y, Z, associated with the celestial body surface (X and Z axes lie in the plane of the landing surface, X axis lies in the plane of the first leg);

2) the spacecraft fixed coordinate system (FCS) $X_cY_cZ_c$, directed along the longitudinal axis of the spacecraft, X_c axis is perpendicular to Y_c axis and lies in the plane XY of the SCS, Z_c completes the right hand triple of the coordinate system;

3) the gravitational coordinate system (GCS) $X_gY_gZ_g$ (Y_g axis is directed along the line of the gravitation force action, X_g axis is perpendicular to Y_g axis and lies in the plane XY of the SCS).

Fig. 1 shows a top view of the spacecraft. Numbers *1*, *2*, *3* refer to contact points between the legs and a celestial body; points P_1, P_2, P_3 refer to locations of the thrusters. Angles γ_2, γ_3 denote a rotation of the second and third legs relative to the axis O_cX_c; points B and D refer to the struts attachment.

The dimensions of all the legs are identical. Fig. 2 shows the main dimensions of the first leg: $l_0(AE)$ — is a projection of the legs AB, AD on the plane X_cY_c; $l_{sw}(AC)$ — is the length of a rod with a shock absorber; l_E — is the distance from the center of mass (CM) of the spacecraft to the point E along the O_cX_c axis; h_0, h_1 — are the distances from the spacecraft CM to the attachment points of the strut and the shock absorber, respectively; H — are the distances from the spacecraft center of mass to the point A along the

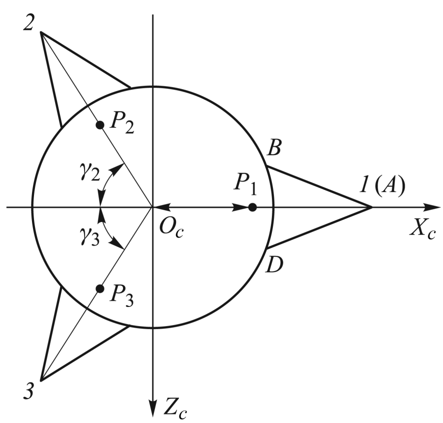

Fig. 1. Top view of the spacecraft with three legs

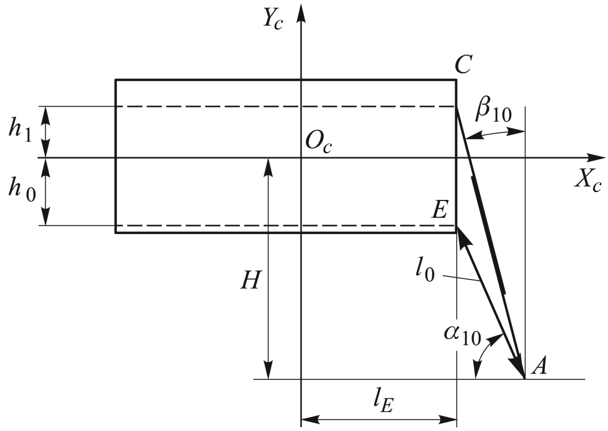

Fig. 2. Main dimensions of the shock absorber and the rod attachment to the spacecraft

O_cX_c axis; α_0 — is the strut inclination angle relative to the line, which is parallel to the O_cX_c axis [7].

Fig. 3 shows the main forces acting on the spacecraft during landing.

P_1, P_2, P_3 — are the thrust forces; mg — is the gravitational force; F_{N1}, F_{N2}, F_{N3} — are the projections of the terrain reaction in the direction of the OY axis of the SCS; F_{T1}, F_{T2X}, F_{T3X} — are the friction forces of the three legs along the OX axis of the SCS.

The following designations are also present in the figure: ϑ — is the spacecraft inclination angle relative to the OXaxis; Θ_g — is the angle between the gravitational vertical and the OY axis of the SCS.

Differential equations describing the longitudinal motion of the spacecraft during landing on the surface of a celestial body along the axes of the SCS of the spacecraft can be defined in the following way [8]:

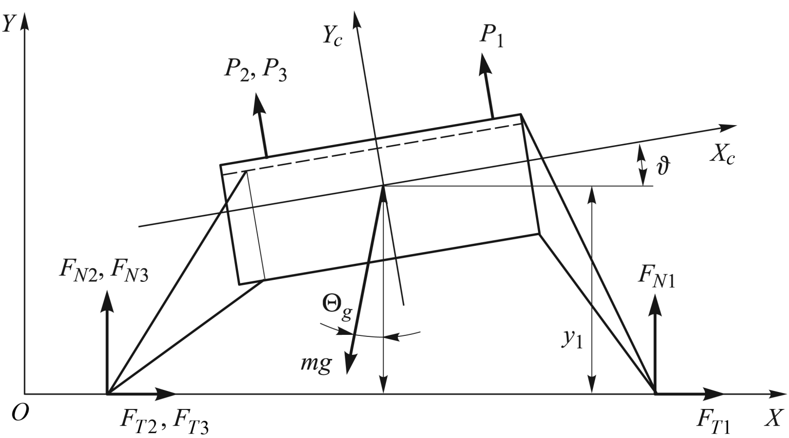

Fig. 3. Forces acting on the spacecraft during landing

$$
\begin{aligned}
&\frac{dV_x}{dt}=\frac{1}{m}\left[F_{T1}+F_{T2X}+F_{T3X}-(P_1+P_2+P_3)\sin\vartheta\right]-g\sin\theta_g;\\
&\frac{dV_y}{dt}=\frac{1}{m}\left[F_{N1}+F_{N2}+F_{N3}+(P_1+P_2+P_3)\cos\vartheta\right]-g\cos\theta_g;\\
&\frac{dx}{dt}=V_x;\\
&\frac{dy}{dt}=V_y;\\
&\frac{d\omega_z}{dt}=\frac{1}{I_z}[(F_{T1}+F_{T2X}+F_{T3X})y+F_{N1}x_1-\\
&\qquad-(F_{N2}+F_{N3})x_2+P_1x_p-P_2x_p\cos\gamma_2-P_3x_p\cos\gamma_3];\\
&\frac{d\vartheta}{dt}=\omega_z.
\end{aligned}
\tag{1}
$$

The legs dimensions h_0, h_1, H, l_E, l_0 — are selected beforehand. The initial values of the following parameters are determined by the formulae:

$$
\begin{aligned}
&\alpha_0=\arcsin\left(\frac{H-h_0}{l_0}\right);\\
&l_{\text{sw}0}=\sqrt{(l_0\cos\alpha_0)^2+(H+h_1)^2};\\
&\beta_0=\arcsin\left(\frac{l_0\cos\alpha_0}{l_{\text{sw}0}}\right).
\end{aligned}
\tag{2}
$$

Let us consider a solution algorithm for the problem of the spacecraft landing on the surface of a celestial body for the first (second) leg, which are directed along the OX axis of the SCS.

The contact condition between the first (second) leg and the surface can be written as follows:

$$\delta y_1=y-y_1=0, \tag{3}$$

here

$$y_{1(2)}=h_0\cos\vartheta-l_E\sin\vartheta+l_0(\alpha_0-\vartheta_{1(2)}); \tag{4}$$

$$\vartheta_2=\arcsin(\cos(\pi-\gamma_2)\sin\vartheta) \tag{5}$$

— is an inclination angle of the spacecraft for the second leg.

For $\delta y_{1(2)}=0$ we find the current values for y and ϑ, which are determined during the integration of equations (1):

$$\alpha_{1(2)}=\arcsin\left(\frac{y+l_E\sin\vartheta-h_0\cos\vartheta}{l_0}\right)+\vartheta_{1(2)};$$

$$H_{1(2)} = l_0 \sin\alpha_{1(2)} + l_E;$$
$$L_{1(2)} = l_0 \cos\alpha_{1(2)} + l_E;$$
$$l_{\mathrm{sw}1(2)} = \sqrt{(H + h_{1(2)})^2 + (L_{1(2)} - l_E)^2};$$

$\delta_{1(2)} = l_{\mathrm{sw}1(2)} - l_{\mathrm{sw}1(2)}$ — is the shock absorber length change;

$$x_1 = l_0 \cos(\alpha_1 - \vartheta) + l_E \cos\vartheta + h_0 \sin\vartheta;$$
$$x_2 = \cos\gamma_2 (l_0 \cos(\alpha_2 - \vartheta_2) + l_E \cos\vartheta_2 + h_0 \sin\vartheta_2);$$
$$\beta_{1(2)} = \arcsin\left(\frac{L_1 - l_E}{l_{\mathrm{sw}1(2)}}\right). \qquad (6)$$

As it is shown in Fig. 4, we can find the shock absorber force by the known δ_1.

$$F_{L1(2)}(\delta) = \begin{cases} 0, \quad \delta_{1(2)} \le \delta_{jk}; \\ F_0 \dfrac{\delta_{1(2)} - \delta_{k1}}{\delta_0}, \quad \delta_{jk} < \delta_{1(2)} < \delta_{k1(2)}; \\ F_0, \quad \delta_{1(2)} \ge \delta_{k1(2)}, \end{cases} \qquad (7)$$

here δ_0 — is an initial value of the flexible deformation region of the shock absorber; δ_{k1j} — is a current value of the flexible deformation region of the shock absorber; δ_{jk} — is an initial value of the deformation during j-th contact between the leg and the terrain, F_0 — is a force during the shock absorber destruction.

Let us formulate the force equilibrium equations in the point A of the first (second) leg along the OX axis and the OY axis. After doing the transformations, we shall obtain the formula for calculating a soil reaction on the leg:

$$F_{1(2)} = F_{L1(2)} \cos(\alpha_{1(2)} + \beta_{1(2)}) / \cos(\alpha_{1(2)} + \mu_{1(2)} - \vartheta), \qquad (8)$$

here $\mu_{1(2)}$ — is a friction coefficient on the first (second) leg.

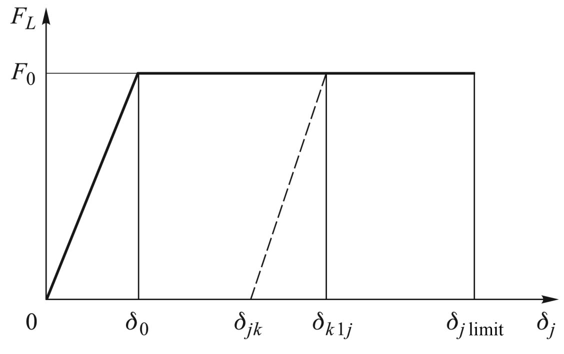

Fig. 4. Shock absorber force characteristics

A soil reaction on the first (second) leg along the OY axis is as follows:

$$F_{N1(2)} = F_{1(2)} \cos \mu_{1(2)}. \tag{9}$$

The friction force magnitude on the second leg is determined

$$F_{T2} = F_2 \sin \mu_2.$$

For determining the friction force on the first (second) leg, we shall calculate its velocity along the OX axis:

$$V_{x1(2)} = V_x + \delta V_{x1(2)}, \tag{10}$$

here V_x — is a velocity of the spacecraft center of mass along the OX axis; $\delta V_{x1(2)} = -V_y(\alpha_{1(2)} - \vartheta)$ — is a velocity of the leg during the shock absorber length change; $\delta V_{x2} = -\delta V_2 \cos \gamma_2$ and $\delta V_{z2} = -\delta V_2 \sin \gamma_2$ — is a displacement projection and a velocity of the second leg along the axes OX and OZ.

The friction force on the first and second legs can be calculated as follows:

$$F_{T1} = F_{T1X} = -F_1 \sin(\mu_1) \frac{V_{x1}}{abs(V_{x1})}. \tag{11}$$

The third leg is symmetrical relative to the second leg; therefore, during the longitudinal motion of the spacecraft landing on the surface of a celestial body, the soil reaction forces on the second and the third legs are identical.

$$F_{N2} = F_{N3}, \;\; F_{TX2} = F_{TX3}. \tag{12}$$

Let us consider an example of using the proposed method for calculating the parameters of the spacecraft longitudinal motion during landing on the surface of the Moon [9, 10].

The design parameters of the spacecraft and the legs are: $m = 900\,\text{kg}$, $I_z = 800\,\text{kg}{\cdot}\text{m}^2$, $h_0 = 0.38\,\text{m}$, $h_1 = 0.20\,\text{m}$, $H = 1.06\,\text{m}$, $l_0 = 0.8\,\text{m}$, $l_E = 1.0\,\text{m}$, $\delta_0 = 0.001\,\text{m}$, $F_0 = 3000\,\text{N}$.

We shall estimate the impact of different forces acting in the shock absorbers during a vertical landing of the spacecraft under the following conditions ($y_0 = 1.4\,\text{m}$, $V_{y0} = 0.1$, $V_{x0} = 0$, $\mu_1 = \mu_2 = 0.2$, $\vartheta_0 = 0°$, $\theta_0 = 0°$). Fig. 5 shows graphs of variance of the soil normal reactions on the legs of the spacecraft at $F_0 = 4000\,\text{N}$ (variant 1), $F_0 = 3000\,\text{N}$ (variant 2), $F_0 = 2000\,\text{N}$ (variant 3).

The graphs of variance analysis showed that in the first case, the soil reaction force on the legs is very high and the strut inclination angle after the spacecraft landing is rather large ($\alpha_\text{f} = 47.8°$). In the third case, the

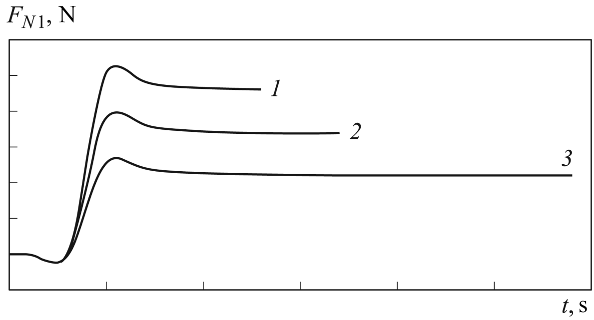

Fig. 5. Graphs of variance of the soil normal reactions on three legs of the spacecraft in the contact points (*1*, *2*, *3*) on the surface of a celestial body at different forces in the shock absorber

soil reaction force on the legs is small but the strut inclination angle after landing is also small ($\alpha_{\mathrm{f}} = 32.3°$). This case is unacceptable since a critical angle of the strut inclination after the spacecraft landing must not exceed $\alpha_{\mathrm{fl}} = 30.0°$. Considering force excitations in the shock absorbers in the third case, the strut inclination angle is lower than the accepted value after the spacecraft landing. Therefore, for the legs of the given dimensions and for the given design parameters of the spacecraft, the second case, where the strut inclination angle after the spacecraft vertical landing is $\alpha_{\mathrm{f}} = 43.4°$, proves to be more acceptable.

We shall estimate different parameters of the spacecraft landing under the following initial conditions: $y_0 = 1.4\,\mathrm{m}$, $V_{y0} = 0.1$, $V_{x0} = 0$, $\mu_1 = \mu_2 = 0.2$, $\vartheta_0 = 10°$, $\theta_g = 0°$. Fig. 6 shows graphs of variance of the soil normal reactions on the spacecraft legs.

It is clear that at the first moment there is a contact between the second and the third legs. Then the spacecraft turns around on these legs until the first leg touches the soil. When the first leg touches the soil, the soil normal reaction force on the second and the third legs decreases to zero two times.

Fig. 7 contains graphs of variance of the deformation of the shock absorbers during the spacecraft landing. The graphs of variance show that

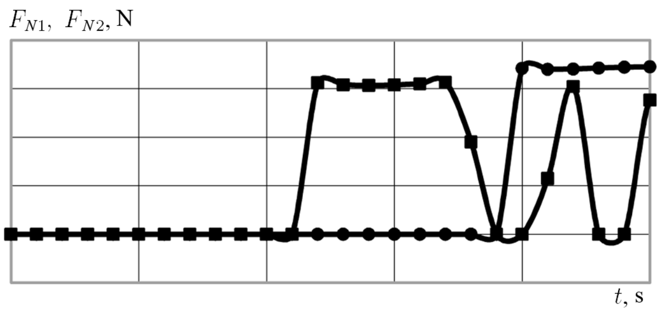

Fig. 6. Graphs of variance of the soil normal reaction forces F_{N1}, F_{N2} on the first (•) and the second (■) legs

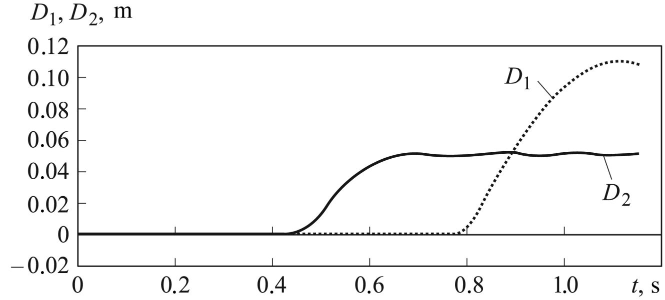

Fig. 7. Graphs of variance of the deformations D_1 и D_2 of the shock absorbers of the first and the second legs of the spacecraft

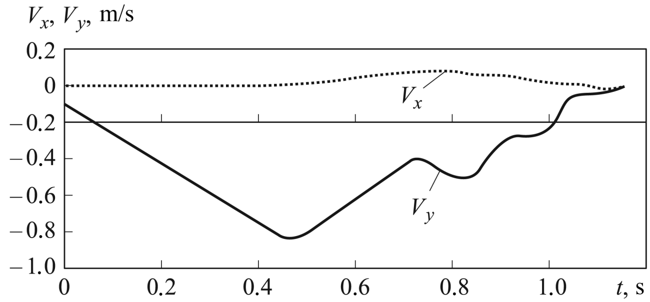

Fig. 8. Graphs of variance of velocities V_x and V_y during the spacecraft landing

the first leg has the largest deformation. The strut inclination angle of the first leg is $\alpha_\text{f} = 33.4°$ after landing.

Fig. 8 shows graphs of variance of the velocities V_x and V_y during the spacecraft landing. It is evident that during the contact of the second and the third legs with the soil, the velocity V_x starts increasing due to a length change of their shock absorbers, while the magnitude of V_y starts decreasing. At the end of the spacecraft landing, these velocities are equal to zero.

Let us note that after the complete landing, the spacecraft inclination angle relative to the surface of the Moon is $\vartheta_\text{f} = -2.87°$.

Conclusions. 1. The article describes a method for estimating some longitudinal motion parameters of a spacecraft during landing on the surface of a small celestial body. The method considers the reaction applied to the movable legs from the surface soil and operation of the thrusters.

2. The authors use the spacecraft landing on the surface of the Moon as an example for estimating the impact of excitations on both the legs position change and forces in the shock absorbers. The authors also consider safe landing conditions: in case of a spacecraft vertical landing, the shock absorber force has a value which allows getting reasonable angles of the struts at moderate soil reaction forces on the legs after landing. The authors discuss the impact of the spacecraft initial inclination relative to the Moon's surface on the dynamics of its motion during landing. The analysis reveals

the legs operation peculiarities, changes of the horizontal and vertical velocities of the spacecraft, and the spacecraft angular position after its landing.

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The original manuscript was received by the editors on 22.05.2013

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The translation of this article from Russian into English is done by A.I. Komissarov, an engineer, Department of Special Machinery, Bauman Moscow State Technical University under the general editorship of N.N. Nikolaeva, Ph.D. (Philol.), Associate Professor, Linguistics Department, Bauman Moscow State Technical University.