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COMPUTATION OF THE STRESS-STRAIN STATE UNDER LOW-CYCLE LOADING USING THE ENDOCHRONIC THEORY OF VISCOPLASTICITY

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Abstract

The term "endochronic" in relation to the theory of plasticity was introduced by K. Valanis in 1971. It was a good marketing ploy. Since then, this term is being widely used. The "endochron" is the "inner time". The concept of "internal variables" was introduced and used much earlier than that proposed by K. Valanis. In a previous article, the author demonstrated results of introducing the endochronic theory to describe complex mechanical behavior of the high-filled polymer material. This paper presents a critical review of theories with the internal variables, as well as new results of using the theory with internal time for the case of low-cycle loading. The stress-strain state of a part after unloading is of particular interest. The function of internal (endochronic) time is determined by the results of testing on tension, compression, shear, shear with the axial compression and then used in the finite element method in solving the task of the stamp indentation into the high-filled polymer material space. Such computation case has practical value. The advantage of introducing the endochronic theory in the finite element method under the low-cycle loading is shown

Keywords

Thermodynamics, internal variables, endochronic theory, high-filled polymer material, low-cycle loading, material testing

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Introduction. At the initial stage of the theory of strength development, the theory of elasticity was most widely used in the engineering structures computation and parts transition to a plastic zone indicated by exhaustion of its bearing capacity. However, the advent of highly loaded systems (nuclear installations, rocket engines, solid propellant charges of the rocket engines, etc.) caused the necessity to introduce the theory of plasticity and creep (viscoplasticity) in computation. The most obvious approach is to use the concept of a yield surface. Relationship between stresses and strains is determined by different ratios in active loading and unloading. At the same time, computation algorithms have complex logic and their implementation requires a lot of machine time. This is especially important for the cyclic and low-cycle loading. For this case, the relay-type equations are used in transition from one ratio to another, when the yield surface is reached. This causes great computation difficulties, especially when using the finite element method (FEM), when the structure is divided into hundreds and sometimes thousands of finite elements.

The concept of internal variables was introduced by V.V. Novozhilov, B. Coleman, L.I. Sedov, I.R. Rice, S. Nemat-Nasser, K. Truesdell [1–6]. Those outstanding engineering scientists tried to bring together the micro and macro approaches to describe mechanical behavior of the materials and used the theory based on the method of non-equilibrium thermodynamics with the internal variables. Internal variables describe the microprocesses at the macro level. Such theories propose a more general mathematical model of the material mechanical behavior and have a more reliable physical basis than the purely phenomeno-logical theories. At the same time, they are quite convenient and simple in use.

Work [1] introduces two macroscopic tensors corresponding to the micro stresses that are caused by plastic deformations and microscopic forces such as dry friction, respectively. According to V.V. Novozhilov, these tensors are the “representatives, messengers” of microstresses. In other words, they are the internal variables.

As far as the author is informed, the term “endochronic (internal time)” in relation to the theory of plasticity was introduced for the first time by K. Valanis [7] in 1971. It was a successful marketing move by K. Valanis, as the term since then was being widely used. We will use this term in the future.

Any elastic process in the internal time scale is instantaneous. Only inelastic deformation processes have a certain duration. Similar to the endochronic time parameter, the thermodynamic time parameter was introduced earlier by A.A. Vakulenko [8].

A student of K. Valanis described his teacher’s theory in detail in a monograph [9], which could only be found in the Library of Congress of the United States.

As far as is known, the author in [10] first pointed out connection between the endochronous theory and the Kadashevich — Novozhilov theory of plasticity [11], which appeared 13 years earlier [7].

The number of hidden (internal) variables is independent from a specific process. It depends on the material properties and the accuracy of the material behavior description. As far as the author is informed, practical application of such an approach and at the same time simplification of the expression for the function of internal time was for the first time indicated in [12]. A more modern application of several internal times is in [13].

The internal variables could be either scalar or tensor. Optimal selection of the comfortable internal parameters and their minimum number, but with enough information about the material reaction, is not a trivial task. The laws of thermodynamics provide only a general approach to finding these internal variables.

The theory of viscoplasticity with the internal parameters was developed by K. Valanis [7] in the most convenient way for practical use. He proposed to use instead of the real time t measured by clock, a dummy internal time z depending on material properties and deformation history. He called the internal time z "endochronic". Therefore, the theory using the parameter z is called the "endochronic time".

The endochronic theory uses the assumption that the material present state depends on all the occurred processes of deformation, but does not depend on the sequence, in which they follow each other. In other words, the entire history of deformation that leads to the same strain is equivalent.

The need for introducing the endochronic time lies in the fact that the internal parameters could not be measured directly during deformation, but researchers are able to measure the components of strain ε_{ij} and temperature T at any point in any time t (like the "black box"). Any increment dt corresponds to the increment in $d\varepsilon_{ij}$ and dT. The three increments would correspond to the increment in the internal time dz dependent of the material properties.

Due to convenience in using the endochronic theory in computation, a large number of papers on this topic were published, see [14, 15], etc.

The endochronic theory was applied not only to describe complex mechanical behavior of the isotropic materials, but was also successfully introduced in computation of the anisotropic materials [16–19], etc. However, this is a subject for a separate discussion.

Data and methods in solving the problems. The previous article by the author [20] demonstrated that the theory of viscoplasticity with an internal time parameter was effectively applied to compute the parts made of material with very unusual mechanical properties, i.e., the highly filled polymer material (HFPM).

This work uses one of the options of the endochronic theory to describe the HFPM mechanical behavior under the low-cycle loading. The stress-strain state of the part after unloading is of particular interest. The material characteristics and unique behavior during loading and unloading are described in article [21].

Specific type of the internal time function for the HFPN [21] material is proposed in [20].

The type of the internal time function and its coefficients is selected from the condition of best description of the test results for the samples made of the HFPM [21].

For the isotropic material, the internal time function is expressed through invariants of stress and strain tensors and has the form given in [20]. All fourteen coefficients for the internal time function are also presented there.

If more than one internal variable is used, the equation for internal time would be simpler. However, the computation process becomes more complicated in this case. Quite a complex type of function of the internal time and a large number of coefficients are a fee for a simple and convenient way to organize the computation process for material with the complex mechanical behavior.

Results. As an example, the problem of re-pressing a rigid stamp into a cylinder from the HFPM was solved. This was a case of loading observed in the real structures. During operation, the low-cycle loading could lead to accumulation of damage and exhaustion of its load-bearing capacity by the structure.

The diameter of the cylinder from HFPM was $2r_0 = 35$ mm, its height — $2h = 70$ mm, stamp diameter — $2R = 10$ mm. The scheme of splitting the object in question into finite elements is given in [20].

The stamp was loaded with a force of $Q = 4.9$ kN in steps (5 loading steps) with organizing the iterative process at each step. Computation process details are in [20]. Then, full unloading was also carried out in steps. After that, the repeated loading was performed. The developed computation program could be used for any number of cycles. The nonlinear system of equations was solved by the additional stresses method. The iterative process at each step was stopped, when in each finite element the difference in the increment in internal time in the previous dz_i and subsequent dz_{i+1} iterations was not exceeding $0.001\,dz_{i+1}$.

Computation results are presented in Fig. 1 and Fig. 2. Figure 1 shows distribution of the axial ε_z (solid) and tangential ε_θ (dashed) in the finite elements column closest to the stamp center during the first loading (*1*) and residual deformations after the full unloading (*2*). Figure 2 shows the axial σ_z (solid) and tangential σ_ϑ (dashed) stresses in megapascals (MPa) under the stamp in the finite elements column closest to the stamp center. Curves *1* correspond to the first loading, curves *2* correspond to the full unloading after the first loading.

The stamp immersion depth at the first loading ($Q = 4.9$ kN) is 1.06 u^e, where u^e is the stamp immersion depth into elastic space under the same load.

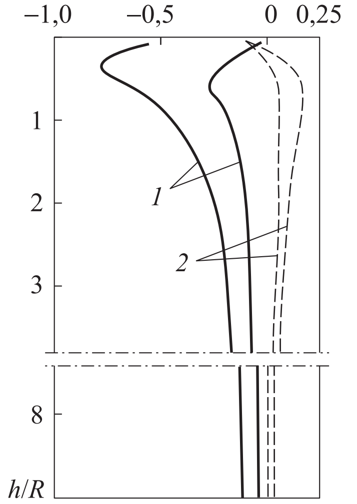

Fig. 1. Distribution of the axial ε_z (solid) and tangential ε_θ (dashed) deformation in % under the stamp in the finite elements column closest to the stamp center. Vertically dimensionless coordinate h / R: curves *1* correspond to the first loading, curves *2* correspond to the full unloading after the first loading

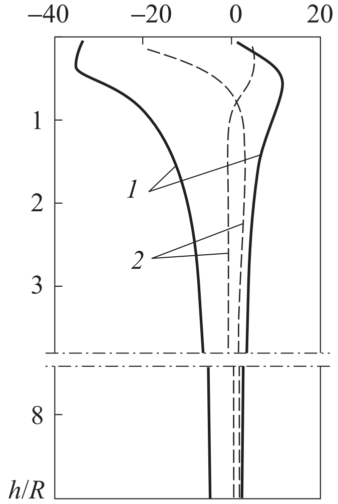

Fig. 2. The axial σ_z (solid) and tangential σ_ϑ (dashed) stresses in megapascals (MPa) under the stamp in the finite elements column closest to the stamp center. Curves *1* correspond to the first loading, curves *2* correspond to the full unloading after the first loading

The immersion residual depth at full discharge is 0.355 u^e. When reloaded, the immersion depth is 1.490 u^e. These results indicate that the force Q magnitude is close to destructive.

Figure 1 shows distribution of the axial ε_z (solid) and tangential ε_θ (dashed) deformation in % under the stamp in the finite elements column closest to the stamp center. Vertically dimensionless coordinate h / R. Curves *1* correspond to the first loading, curves *2* correspond to the full unloading after the first loading.

Figure 2 shows the axial σ_z (solid) and tangential σ_ϑ (dashed) stresses under the stamp in the finite elements column closest to the axis at load $Q =$ $= 4.9$ kN. Curves *1* correspond to the first loading, curves *2* correspond to the full unloading after the first loading.

Discussion of the results obtained. The results obtained clearly show, how the damage accumulation process occurs under the low-cycle loading. Computation was carried out at a very significant load ($Q = 4.9$ kN). With the selected stamp size, this is a large load. Such dimensions of the stamp simplify the experiment [20]. In turn, computation under such a load most clearly demonstrates the residual deformation accumulation process already during the second loading cycle. The developed program makes it possible to compute and evaluate the residual deformation at any number of the loading cycles.

Conclusion. Based on the results of this paper, it could be concluded that the theory of viscoplasticity with the internal time parameter provides good results in computing machine parts made of materials with the complex mechanical behavior not only under simple loading, but also under the low-cycle loading. A computation program based on the endocrine theory is designed to calculate the stress-strain state only under the active loading, and it is upgraded to a program for the low-cycle loading only by adding one operator that determines alteration in the load increment sign. Introducing the approach proposed in this paper makes it possible to determine the residual deformations accumulation after unloading, which could eventually lead to a structural failure. The damage accumulation process is very important, especially in the highly loaded structures.

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