UDC 629.7 DOI: 10.18698/0236-3941-2021-2-4-15

LOCAL FORM OF LOSS OF STABILITY OF HONEYCOMB ENERGY ABSORBER PLATES

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Abstract

Landing devices perform the energy absorption function during the spacecraft motion. Precise analytical solution to the problem of stability of a lengthy plate with free edge exposed to the edge compressive load was obtained in order to analyze operation of the honeycomb materials used in structural elements absorbing the moving bodies energy. General solution analysis of the differential equation for a lengthy plate bending in the deflected position was carried out, and general solution is subjected to the boundary conditions corresponding to the loaded free edge. Critical load value and form of the loss of stability were determined. Critical load identified value was significantly lower than the critical load for a plate supported on the loaded edge. The loss of stability identified form was characterized by sharp deflection localization near the loaded edge and could create conditions for forming a local fold near the loaded edge. Obtained analytical solution was verified by comparing it with results of the similar numerical solution. Comparison performed revealed satisfactory agreement both in the critical load value and in the form of loss of stability for two solutions obtained by different methods. The results obtained could be used in designing energy absorbers made of honeycomb materials, as well as in other areas of technology

Keywords

Motion energy absorption, honeycomb material, compressed plate stability

Received 26.03.2020
Accepted 15.09.2020

Introduction. The problem of a moving object deceleration is encountered in design and development of landing devices for spacecraft [1], road damping barriers [2] and in other areas of technology [3]. Honeycomb material [2, 4, 5], for example, the HexWeb® brand (Hexcel, https://www.hexcel.com) material, is used for absorbing the kinetic energy. Deformable energy absorbers made of honeycomb material were used in the Apollo project lunar module providing soft landing on the Lunar surface [6], as well as in the unmanned spacecraft landing devices [4].

Honeycomb energy absorber stiffness characteristic under compression along the direction parallel to the lateral faces of hexagonal prisms-tubes (T direction in Fig. 1 or "tubular" direction) provided by manufacturers is close to optimal, i.e., deformation force remains practically constant with an increase in displacement (Fig. 2) [1].

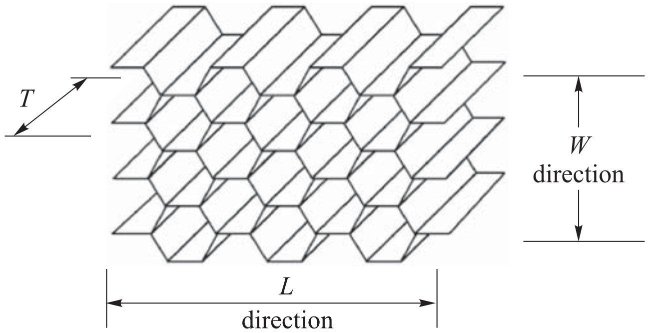

Fig. 1. Hexagonal honeycomb design diagram [4]

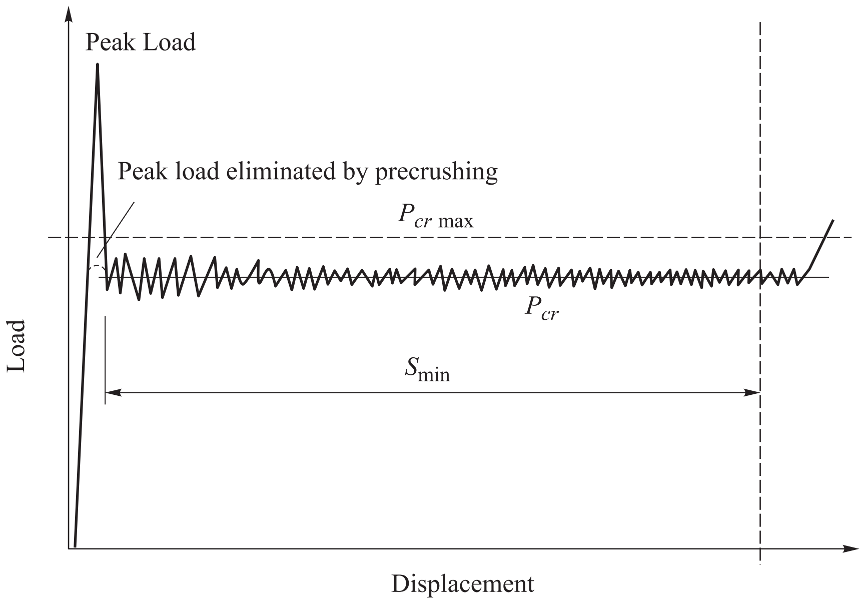

Fig. 2. HexWeb® honeycomb material load-displacement characteristic [4]

This *work objective* is to study the causes of honeycomb energy absorbers deformation under a load close to constant. Part of the honeycomb (1/6 part in hexagonal honeycombs) is considered as a lengthy plate loaded on the edge by a compressive force. Analytical solution is widely known on stability of a compressed lengthy plate with all edges having free rigid supports that prevent displacement perpendicular to the median surface. In this case, the bending deformation area corresponding to the loss of stability form extends over the entire plate length repeating the shape with a step equal to the plate width [7, 8].

In the case of honeycomb material, longitudinal edges are unable to move due to the structure symmetry. Thus, the loaded edge is free from displacement restriction in the direction perpendicular to the median surface. Design schemes containing a loaded free edge are missing in the well-known monographs and reference books on the stability of compressed plates [7, 9–13]. This is explained by the fact that the loaded plate edge in real load-bearing structures is reinforced with a stiffening rib. The problem of determining critical load for a plate with a free edge was considered in the master's thesis [14]. However, solution was not completed. This work provides a complete precise solution to the problem of stability of a lengthy plate with free edge exposed to edge compression load. Critical load values and loss of stability form characterized by sharp localization of deflections near the loaded edge were determined.

Obtained analytical solution was verified by comparing it with results of a similar numerical solution.

Loss of stability local form obtained makes it possible to explain constancy of the honeycomb material deformation force during compression in the direction parallel to the hexagonal prisms-tubes lateral faces (T direction in Fig. 1).

Stability study of the lengthy plate with a free edge using the differential equilibrium equation for a plate element. Let us consider a rectangular plate with the h thickness, which length is significantly greater than the b width, loaded with the T compression force uniformly distributed along one of the short edges and directed parallel to the lengthy edges (Fig. 3). Unloaded edges are pivotally supported. Loaded edge displacements are not limited.

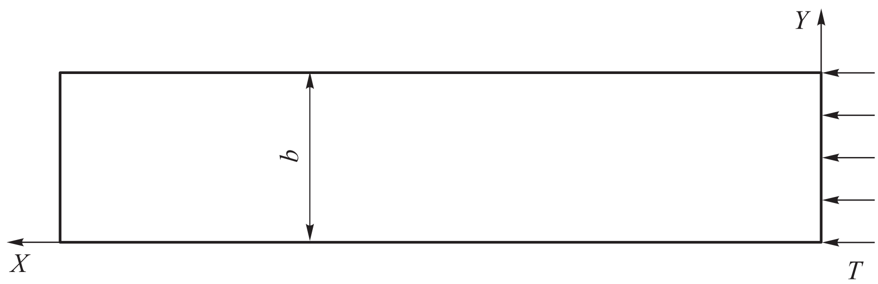

Fig. 3. Design scheme of a lengthy plate, loaded compression layer applied to the short edge

The XY plane coincides with the plate undeformed median surface. Differential equilibrium equation for the bent plate element [7] has the following form:

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+T\frac{\partial^2 w}{\partial x^2}=0, \qquad (1)$$

where $D=\dfrac{Eh^3}{12\,(1-\nu^2)}$ is the cylindrical stiffness; E is the Young's modulus; ν is the Poisson's ratio.

Let us look for displacements in the direction perpendicular to the w median surface (deflections) in the following form:

$$w(x,\, y)=f(x)\sin\left(\frac{\pi y}{b}\right). \qquad (2)$$

Substituting (2) into (1), we obtain

$$\frac{\partial^4 f}{\partial x^4}+\left(\frac{T}{D}-2\left(\frac{\pi}{b}\right)^2\right)\frac{\partial^2 f}{\partial x^2}+\left(\frac{\pi}{b}\right)^4 f=0. \qquad (3)$$

Characteristic equation will have the following form [15]:

$$r^4+pr^2+q=0, \qquad (4)$$

where $p=\dfrac{T}{D}-2\left(\dfrac{\pi}{b}\right)^2$ and $q=\left(\dfrac{\pi}{b}\right)^4$. Solution to the biquadratic equation (4) is as follows:

$$r^2=-\frac{p}{2}\pm\sqrt{\left(\frac{p}{2}\right)^2-q}.$$

Let us consider the case, when the radical expression is equal to zero:

$$\frac{1}{4}\left(\frac{T}{D}-2\left(\frac{\pi}{b}\right)^2\right)^2-\left(\frac{\pi}{b}\right)^4=0;$$

$$\frac{T}{D}=4\left(\frac{\pi}{b}\right)^2; \qquad (5)$$

$$T=4D\left(\frac{\pi}{b}\right),$$

where $r=\pm\sqrt{\left(\dfrac{\pi}{b}\right)^2}\,i.$

General solution of the differential equation (3) takes the following form:

$$f(x)=C_1\sin\left(\frac{\pi x}{b}\right)+C_2\cos\left(\frac{\pi x}{b}\right).$$

If $C_2 = 0$, which corresponds to the fulfillment of the boundary conditions for hinge support on the loaded edge

$$f(x) = C\sin\left(\frac{\pi x}{b}\right).$$

In this case, the loss of stability form will take the following form:

$$w(x, y) = C\sin\left(\frac{\pi x}{b}\right)\sin\left(\frac{\pi y}{b}\right). \tag{6}$$

The plate is divided into square fields, where displacements are periodically repeated as the *x* coordinate increases. Dependencies (5) and (6) correspond to the known solution for a plate hinged along all the edges [7, 8].

In order to determine the loss of stability form corresponding to the absence of support on the loaded edge, let us consider the $\frac{1}{4}\left(\frac{T}{D} - 2\left(\frac{\pi}{b}\right)^2\right) - \left(\frac{\pi}{b}\right)^2 < 0$ case, where $T < 4D\left(\frac{\pi}{b}\right)^2$.

Let us find the plate loss of stability form corresponding to a lower value of the critical force than in the hinged support on a loaded edge. Then, changing the radical expression sign for r^2, we obtain

$$r^2 = -\frac{1}{2}\left(\frac{T}{D} - 2\left(\frac{\pi}{b}\right)^2\right) \pm \sqrt{\left(\frac{\pi}{b}\right)^4 - \frac{1}{4}\left(\frac{T}{D} - 2\left(\frac{\pi}{b}\right)^2\right)^2}\, i,$$

or (see [15])

$$r^2 = \overline{x} \pm \overline{y}i = \rho\cos\varphi.$$

Complex number modulus is $\rho = \sqrt{\overline{x}^2 + \overline{y}^2}$, or $\rho = \left(\frac{\pi}{b}\right)^2$.

Cosine of the complex number argument

$$\cos\varphi = \frac{\overline{x}}{\rho} = -\left(\frac{T}{2D}\left(\frac{b}{\pi}\right)^2 - 1\right).$$

Then (see [15])

$$r = \sqrt{r^2} = \frac{\rho}{2}\cos\frac{\varphi}{2} = \alpha + \beta i;$$

$$\cos\frac{\varphi}{2} = \sqrt{\frac{1}{2}\left(1 + \cos\varphi\right)} = \sqrt{\left(1 - \frac{T}{4D}\left(\frac{b}{\pi}\right)^2\right)}.$$

Sine of the half argument

$$\sin\frac{\varphi}{2}=\sqrt{\frac{1}{2}\left(1-\cos\varphi\right)}=\frac{b}{2\pi}\sqrt{\frac{T}{D}}.$$

The r real part

$$\sqrt{\rho}\cos\frac{\varphi}{2}=\sqrt{\left(\left(\frac{\pi}{b}\right)^2-\frac{T}{4D}\right)}=\alpha.$$

The r imaginary part

$$\sqrt{\rho}\sin\frac{\varphi}{2}=\frac{1}{2}\sqrt{\frac{T}{D}}=\beta.$$

General solution of the differential equation (3) takes the following form [15]:

$$f(x)=e^{-\alpha x}\left(C_1\sin\beta x+C_2\cos\beta x\right)+e^{\alpha x}\left(C_3\sin\beta x+C_4\cos\beta x\right).$$

Solution obtained should be subordinated to boundary conditions of the problem under consideration. From the condition of limited deflections of the arbitrarily lengthy plate, it follows that $C_3=C_4=0.$ Then

$$f(x)=e^{-\alpha x}\left(C_1\sin\beta x+C_2\cos\beta x\right). \tag{7}$$

Thus, expression for deflections (2) takes the following form:

$$w(x,y)=e^{-\alpha x}\left(C_1\sin\beta x+C_2\cos\beta x\right)\sin\left(\frac{\pi y}{b}\right). \tag{8}$$

Let us write down the two boundary conditions at the loaded edge with $x=0$ in the following form [16]:

$$M_x=0;$$

$$Q_x-\frac{\partial M_{xy}}{\partial y}=-T\frac{\partial w}{\partial x}.$$

Replacing internal forces with their expressions through deflection, we get

$$\begin{gathered}\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}=0;\\ \frac{\partial^3 w}{\partial x^3}+(2-\nu)\frac{\partial^3 w}{\partial x\partial y^2}=-\frac{T}{D}\frac{\partial w}{\partial x}.\end{gathered} \tag{9}$$

Substituting (7) into (9), two linear homogeneous equations are obtained for C_1 and C_2:

$$-2C_1\alpha\beta + C_2\left(\alpha^2 - \beta^2\right) - \nu\left(\frac{\pi}{b}\right)^2 C_2 = 0;$$

$$C_1\left(3\alpha^2\beta - \beta^3\right) + C_2\left(3\alpha\beta^2 - \alpha^3\right) - \left(\frac{\pi}{b}\right)^2 (2-\nu)\left(C_1\beta - C_2\alpha\right) =$$

$$= -\frac{T}{D}\left(C_1\beta - C_2\alpha\right).$$

Grouping expressions at constants, let us find

$$-2C_1\alpha\beta + C_2\left(\alpha^2 - \beta^2 - \nu\left(\frac{\pi}{b}\right)^2\right) = 0;$$

$$C_1\left(3\alpha^2\beta - \beta^3 - \left(\frac{\pi}{b}\right)^2 (2-\nu)\beta + \frac{T}{D}\beta\right) +$$

$$+ C_2\left(3\alpha\beta^2 - \alpha^3 + \left(\frac{\pi}{b}\right)^2 (2-\nu)\alpha - \frac{T}{D}\alpha\right) = 0.$$

Equating the determinant of a given homogeneous system to zero, we obtain

$$-2\alpha^2\beta\left(3\beta^2 - \alpha^2 + \left(\frac{\pi}{b}\right)^2 (2-\nu) - \frac{T}{D}\right) -$$

$$-\beta\left(\alpha^2 - \beta^2 - \nu\left(\frac{\pi}{b}\right)^2\right)\left(3\alpha^2 - \beta^2 - \left(\frac{\pi}{b}\right)^2 (2-\nu) + \frac{T}{D}\right) = 0. \qquad (10)$$

Let us introduce the following notation:

$$t = \frac{T}{D}\left(\frac{b}{\pi}\right)^2. \qquad (11)$$

Then

$$\alpha^2 = \left(\frac{\pi}{b}\right)^2 - \frac{T}{4D} = \left(\frac{\pi}{b}\right)^2\left(1 - \frac{t}{4}\right);$$

$$\beta^2 = \frac{T}{4D} = \left(\frac{\pi}{b}\right)^2 \frac{t}{4};$$

$$\alpha^2 - \beta^2 = \left(\frac{\pi}{b}\right)^2\left(1 - \frac{t}{4}\right).$$

In this case, (10) takes the following form:

$$-2\left(1-\frac{t}{4}\right)\left(3\frac{t}{4}-\left(1-\frac{t}{4}\right)+(2-\nu)-t\right)-$$
$$-\left(1-\frac{t}{2}-\nu\right)\left(3\left(1-\frac{t}{4}\right)-\frac{t}{4}-(2-\nu)+t\right)=0.$$

Let us consistently carry out identical transformations

$$-2\left(1-\frac{t}{4}\right)(1-\nu)-\left(1-\frac{t}{2}-\nu\right)(1+\nu)=0.$$

We finally get

$$t=3-2\nu-\nu^2.$$

Taking into account (11), expression for the critical load is obtained

$$\frac{T}{D}=\left(\frac{\pi}{b}\right)^2\left(3-2\nu-\nu^2\right), \quad (12)$$

for $\nu=0.3$

$$T=2.31D\left(\frac{\pi}{b}\right)^2. \quad (13)$$

Substituting (12) into any of equations (8), relation between the constants is obtained

$$C_1=C_2\frac{\nu-1}{\sqrt{\left(3-2\nu-\nu^2\right)}}. \quad (14)$$

Using the identity transformations from (12), we obtain

$$\left(\frac{\pi}{b}\right)^2-\frac{T}{4D}=\left(\frac{\pi}{b}\right)^2\left(\frac{1+\nu}{2}\right)^2. \quad (15)$$

Substituting (12), (14) and (15) into (8), the plate loss of stability form is obtained

$$w(x,\ y)\ =C_2e^{-(1+\nu)\xi}\left(\frac{\nu-1}{\mu}\sin\xi\mu+\cos\xi\mu\right)\sin\psi, \quad (16)$$

where $\mu=\sqrt{3-2\nu-\nu^2}$; $\xi=\frac{x}{2}\frac{\pi}{b}$; $\psi=\frac{\pi y}{b}$.

Verification of the obtained analytical solution by comparison with results of the numerical calculations. In order to verify the obtained analytical solution, numerical calculation of stability of an aluminum alloy plate 200 mm wide, 2 mm thick and 1000 mm long was performed. Figs. 4 and 5 demonstrate the loss of stability form according to the results of analytical and numerical solutions.

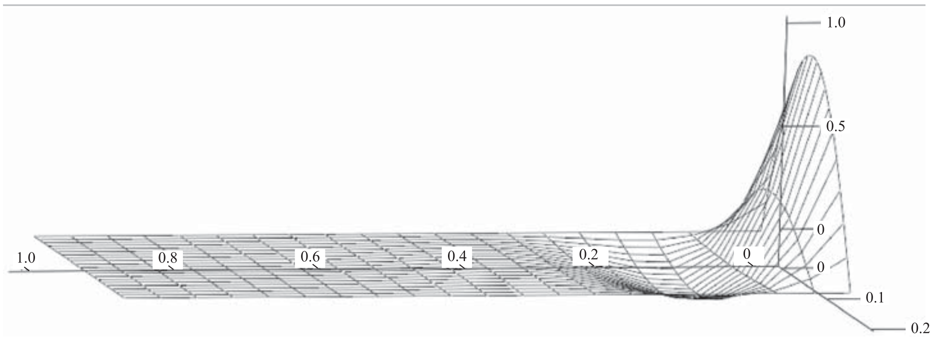

Fig. 4. Loss of stability form according to the results of analytical solution (16)

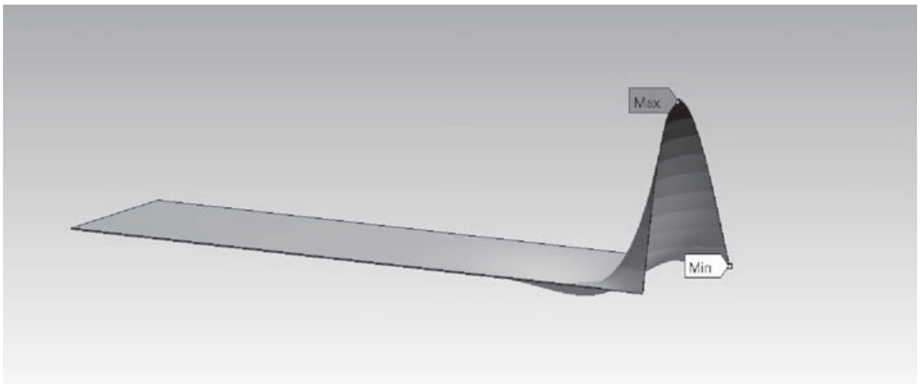

Fig. 5. Loss of stability form according to the results of numerical calculation

Comparison of the results of analytical and numerical solutions shows their satisfactory correspondence. Discrepancy in the critical load value was ~ 1.7 % when using a grid with square shell elements at 10 elements across the plate width. Loss of stability form is presented in Figs. 4 and 5.

Let us compare the result obtained with the known solution for a plate hingedly supported along all the edges. Fig. 6 shows the loss of stability form in accordance with dependence (6) of a plate with the same dimensions and of the same material as in Fig. 4. In this case, the form has a cyclically repeating structure with constant amplitude along the lengthy edge.

The form in Figs. 4 and 5 differs from the form in Fig. 6 by significant localization of deflections near the edge, where the load is applied (in all the Figures, the right edge is loaded).

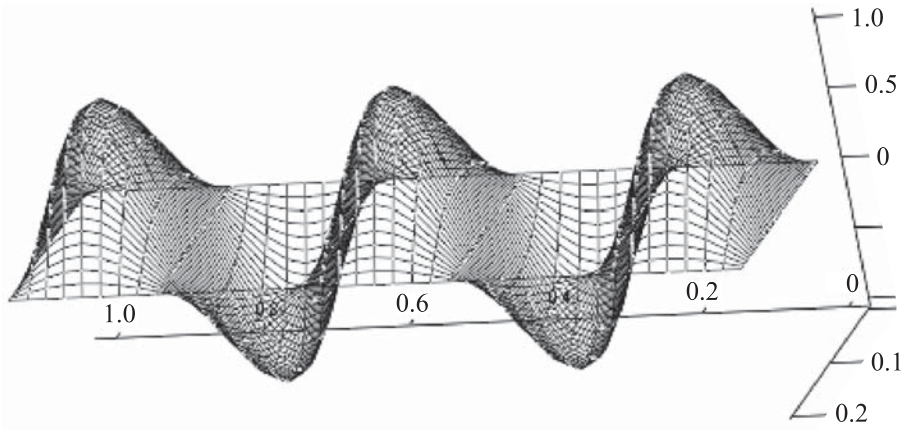

Fig. 6. Loss of stability form according to the analytical solution results (6) for the case of the loaded edge hinged support

Considering these results, the following conclusions could be made on the nature of deformation in a plate with a free edge: loss of stability form is determined by a single geometric parameter, i.e., the plate width; lateral displacements are local and are rapidly decreasing with distance from the plate loaded edge; critical load is by $\left(\frac{2.31}{4}\right)$ times lower than for the case of a hingedly supported loaded edge, see formulas (5), (12) and (13).

Findings on the nature of honeycomb elements deformation using the obtained solution results. Based on the obtained solution results, an explanation of the cellular material deformation mechanism with a constant force could be presented and is as follows.

Local loss of stability form creates prerequisites for formation of a fold adjacent to the loaded edge. Its local deformation continues during plastic deformations with formation of a fold near the loaded edge.

Local fold at the plate edge contributes to the next fold formation with further deformation. In this case, formation force of each subsequent fold would be of the order of the plate loss of stability force with a free edge. Consequently, deformation force accompanied by the folds sequential formation would be approximately constant. Honeycomb material appears to be a set of plates, and it would be deformed by almost constant force (see Fig. 2). The plates local loss of stability form determines this feature of the honeycomb material deformation process.

Conclusion. A new solution was obtained to the problem of stability of a lengthy plate loaded with compression force applied to the free short edge. Critical load value was determined, and loss of stability form was found, which characteristic feature is the deflection localization near the loaded edge. Verification of the obtained analytical solution was carried out by comparingit with numerical calculation.

Established deformation features of a lengthy plate with free edge upon loss of stability were used to analyze behavior of the honeycomb material exposed to compression in the pipe direction in the material absorbing the energy of colliding bodies.

Results obtained could be used in designing the honeycomb energy absorbers, as well as in other areas of technology, where the considered design scheme is applicable.

Translated by K. Zykova

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Please cite this article as:

Uvarov A.I. Local form of loss of stability of honeycomb energy absorber plates. *Herald of the Bauman Moscow State Technical University, Series Mechanical Engineering*, 2021, no. 2 (137), pp. 4–15. DOI: https://doi.org/10.18698/0236-3941-2021-2-4-15