UDC 531.76+534.134+534.141.2 DOI: 10.18698/0236-3941-2019-6-89-104

CALCULATED-EXPERIMENTAL DETERMINATION OF VALUES NATURAL FREQUENCIES OSCILLATIONS OF THE CASE AND PARAMETERS OF THE CASE DETERMINING THESE VALUES

A.V. Proskurin niik@vniitf.ru
A.V. Shlishevskiy niik@vniitf.ru

Russian Federal Nuclear Center — Zababakhin All-Russian Scientific Research Institute of Technical Physics, Snezhinsk, Chelyabinsk Region, Russian Federation

Abstract

The task of studying the behavior of various structures under the influence of intense pulsed (shock) loads that arise during the operation of many modern structures, machines and devices remains relevant for many years. One of the stages of this study is to find the values of the natural frequencies of vibrations and design parameters that determine these values. To find the natural frequencies of oscillations of the body, which are four flanges connected by cylindrical shells, the finite element method and analytical solutions were used. An analysis of experimental studies. Determination of natural frequencies values and design parameters determining these values will allow evaluating the response of the structure to the loads acting during its operation, and making corrections to the structure that change the response of the structure, as well as will help to more accurately select the installation sites of piezoelectric accelerometers for recording drums accelerations

Keywords

Natural frequency of oscillations, form of natural oscillations, shock loading, spectral density, experimental studies, piezoelectric accelerometer

Received 10.12.2018

Introduction. In these works [1–4] the experience of using modal analysis on the basis of FEM (finite element method) in comparison with experimental data for verification of computational models, as well as for identification of natural frequencies and waveforms of different structures is presented. The main disadvantage of these works is that the computational and experimental studies are given without determining the parameters affecting the eigenfrequencies and shapes of the structure oscillations, which in some problems can be of practical interest. In shock [5] and vibro-shock tests of structures, the practical interest is to assess the possibility of influencing the reaction of the structure under mechanical impact, that is, to determine the parameters of the structure affecting its reaction. This paper is dedicted to this issues.

Object of research. The body was chosen as the object of research (Fig. 1), which has practical interest in shock and vibration tests of structures. Corpus made of aluminum alloy D16 (Д16) with mechanical characteristics: density ρ = 2640 kg/m^3, Young's module $E = 0.72 \cdot 10^{11}$ Pa, Poisson's ratio $\mu = 0.33$ [6–9]. Steel threaded bushings are screwed into the holes. Overall dimensions of the case: radius of the circumscribed circle 290 mm, length 540 mm.

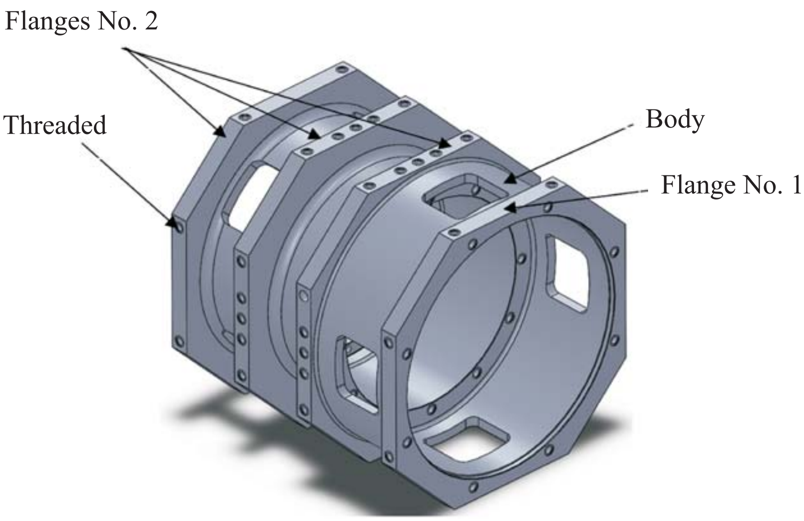

Fig. 1. 3D body model

As seen in Fig. 1, the housing is a structure consisting of octagonal reinforcing flanges connected by cylindrical shells with rectangular windows. Two types of reinforcing flanges are used in the casing, differing in the magnitude of the inner diameter, one flange has an inner diameter d = 472 mm (flange No. 1), and three — d = 427 mm (flange No. 2). Flange thickness h = 35 mm.

The objective of the study is to determine the values of the natural frequencies of oscillations and the parameters of the case, affecting these values. The first eight frequencies of the body natural vibrations were determined.

Computational studies of the natural frequencies of the body. We will determine the natural frequencies of the case using the FEM of the APM finite element analysis package (registration number No. 330 of the certification certificate of the APM Structure 3D software, © STC APM) [10].

The calculation was carried out using the module for calculating the natural frequencies, with free boundary conditions of the case.

In the first calculations, the impact of accounting for the presence of threaded bushings, holes for threaded bushings, chamfers and roundings on the natural frequencies of the housing was evaluated. The difference in the values of the natural frequencies of oscillations of the housing without holes for threaded bushings, chamfers and fillets and the housing with bushings, chamfers and fillets was not more than 6 %. Taking into account the insignificance of the difference in frequencies, further calculations were carried out without taking into account the presence of holes for threaded bushings, chamfers and fillets, which significantly reduced the requirements for computing resources. The finite element mesh was generated from hexagonal elements with an edge size of 10 mm. As a result of mesh generation, the model was divided into 44621 elements. After the grid was created, its quality was assessed, as a result of which it was determined that the finite element model consists mainly of 20 nodal hexagonal elements HEX20, which is an acceptable result.

The values of the first eight eigenfrequencies of the case calculated using the FEM are given in Table 1 (second column). In Fig. 2–8 show the vibration modes of the case at these frequencies.

From the above figures it follows that one part of the natural vibration frequencies of the body is determined by the vibrations of one of its elements, for example, one of the flanges (Fig. 2–5, 7, 8), while the other body elements are practically not deformed. The other part of the frequencies of natural vibrations is determined by the vibrations of several elements of the body (Fig. 6).

Finding body parameters that determine the values of the frequencies of natural vibrations. To find the parameters that determine the eigenfrequencies of the vibration of the structure under consideration, we replace its structural elements with elements that are simpler in form, for which analytical dependencies are found that determine the natural vibration frequencies, such as circular rings, square and round plates, and a cylindrical shell. We begin our consideration with circular rings. As the outer diameter of the ring, we take the average value of the diameter of the circumscribed circle and the diameter of the inscribed circle in the octagon. Thus, the rings have an outer diameter $D = 550$ mm, inner diameters $d = 472$ mm (the ring No. 1 for the flange No. 1) and $d = 427$ mm (the ring No. 2 for the flange No. 2), thickness $h = 35$ mm.

It is known [11] that a circular ring can perform several types of vibrations: tensile-compression vibrations, torsional vibrations, bending vibrations in the plane of the ring and bending vibrations consisting of displacements perpendicular to the plane of the ring and torsion.

Table 1

Natural frequencies and vibration modes of the case and the corresponding natural frequencies of the rings

No.	Natural frequencies of the case obtained using FEM, Hz	No. Fig.	Waveform of simplified elements	Frequencies obtained by analytical solutions for simplified elements, Hz	Peaks spectrum, Hz
1	463.93	2	The first shape of bending vibrations in the plane of the ring No. 1	385	450–501 Flange No. 1
2	505.58	3	First shape bending vibrations from the plane of the ring No. 1	334	450–501 Flange No. 1
3	843.78	4	The first shape of bending vibrations in the plane of the ring No. 2	664	740–878 Flange No. 2
4	908.77	5	The first shape of bending vibrations from the plane of the ring No. 2	366	740–878 Flange No. 2
5	1016	6	Oscillations of a round plate (flange No. 1 and flange No. 2) $n = 2$, $s = 0$, $\alpha = 5.251$. Oscillations of a square plate (flange No. 1 and flange No. 2) with two nodal lines passing through the midpoints of the sides $\alpha = 14.1$	783/526 (715/480)*	1073–1080 Flange No. 1
6	1074.3	7	The second shape of bending vibrations in the plane of the ring No. 1	1088	1073–1080 Flange No. 1
7	1074.4				
8	1081.5	8	Radial vibrations of a cylindrical shell ($n = 1$, $q = 2$)	1147	1073–1080 Flange No. 1

* — in parentheses are the frequency values for the dimensions of flange No. 1.

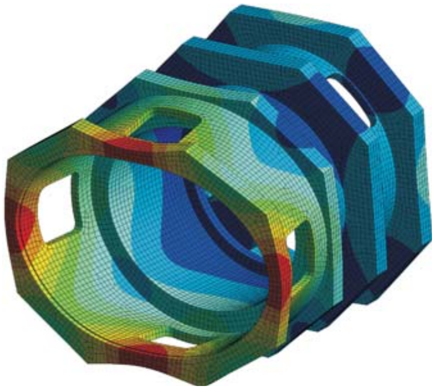

Fig. 2. f = 463.93 Hz

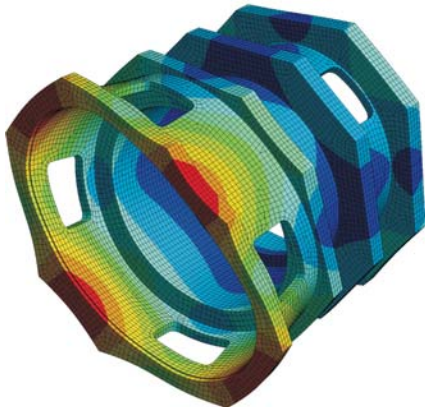

Fig. 3. f = 505.58 Hz

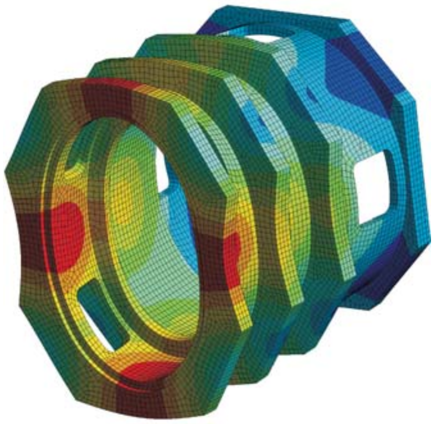

Fig. 4. f = 843.78 Hz

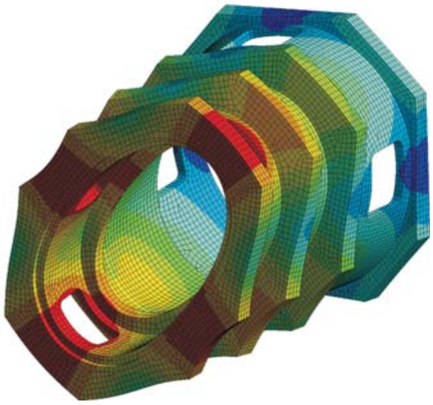

Fig. 5. f = 908.77 Hz

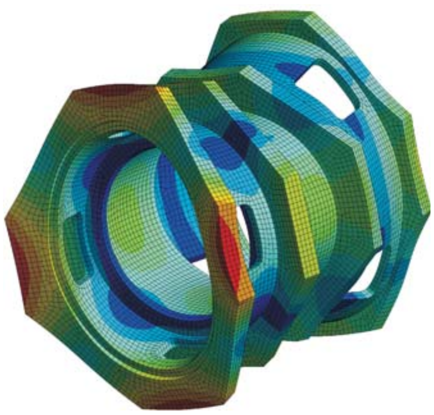

Fig. 6. f = 1016 Hz

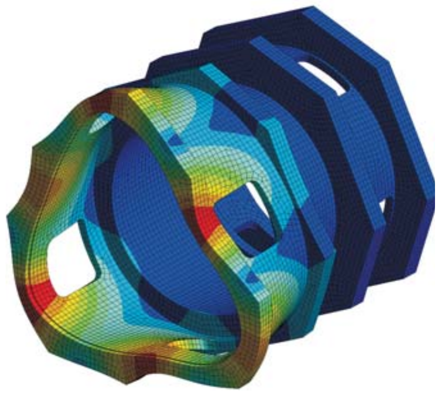

Fig. 7. f = 1074.3 Hz

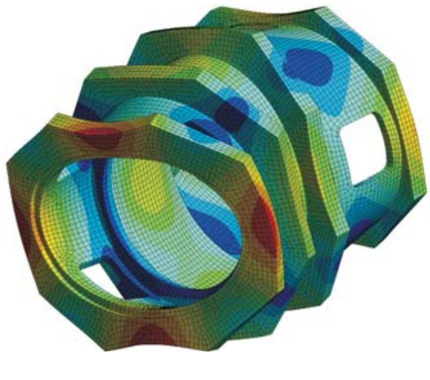

Fig. 8. f = 1081.5 Hz

The vibrational frequencies of tension-compression are determined, according to [11], by the formula

$$f_{\text{tens-comp}\ i} = \frac{a}{2\pi r}\sqrt{\left(1+i^2\right)}, \quad i = 0, 1, 2, 3, \ldots, \tag{1}$$

where $a = \sqrt{E/\rho}$ is sound velocity in the material of the ring; r is radius of the center line of the ring; E is elastic module; ρ is density, i is the number of half-waves located in a circle.

Torsional vibrations of the ring are determined according to [11], by the formula

$$f_{\text{tors}\ i} = \frac{a}{2\pi r}\sqrt{\left(1+i^2\right)\frac{J_x}{J_p}}, \quad i = 0, 1, 2, 3, \ldots, \tag{2}$$

where $J_x = \dfrac{(D-d)h^3}{24}$ is the moment of inertia of the cross section relative to its main axis parallel to the plane of the ring; $J_z = \dfrac{(D-d)^3 h}{96}$ is the moment of inertia of the cross section relative to its main axis perpendicular to the plane of the ring; J_p is the polar moment defined as $J_p = J_x + J_z$.

Bending vibrations in the plane of the ring are determined according to [11], by the formula

$$f_{\text{bend1}\ i} = \frac{a}{2\pi r}\sqrt{\frac{J_z}{Sr^2}\frac{i^2\left(1-i^2\right)^2}{\left(1+i^2\right)}}, \quad i = 2, 3, \ldots, \tag{3}$$

where $S = \dfrac{D-d}{2}h$ is ring cross-section area.

Bending vibrations, including both displacements at right angles to the plane of the ring and torsion, are determined according to [11] by the formula

$$f_{\text{bend2}\ i} = \frac{a}{2\pi r}\sqrt{\frac{J_x}{Sr^2}\frac{i^2\left(i^2-1\right)^2}{\left(i^2+1+\mu\right)}}, \quad i = 2, 3, \ldots, \tag{4}$$

where μ is Poisson's ratio.

The calculations performed by formulas (1)–(4) showed that the lowest vibrational frequencies of the rings are determined by the first two for the ring No. 1 and the first for the ring No. 2 forms of bending vibrations in the plane of the ring and the first forms of bending vibrations consisting of displacements perpendicular to the plane of the ring (334, 385, 1088 Hz is the ring No. 1; 366, 664 Hz is the ring No. 2).

Oscillations of flanges can also be considered as oscillations of plates with edges that are free along the entire contour. The natural frequency of the plate is determined [11, 12] by the formula

$$f_{\text{plate}} = \frac{1}{2\pi}\frac{\alpha}{r^2}\sqrt{\frac{D_{\text{sylin}}}{\rho h}}, \tag{5}$$

where $D_{\text{sylin}} = \dfrac{Eh^3}{12\left(1-\mu^2\right)}$ is the bending stiffness of the plate, h is the plate thickness, r is the characteristic plate size (radius for a round plate, side size for a square plate), α is a constant depending on the shape of the vibrations. For a free round plate of radius $r = \dfrac{550+427}{4} = 244.2$ mm and h = 35 mm (flange No. 2) and the mode of vibration with two nodal diameters (n = 2) and without nodal circles (s = 0) α = 5.251, and respectively the oscillation frequency is $f_{\text{round plate}} = 783$ Hz ($f_{\text{round plate}} = 715$ Hz for the flange No. 1 $r = \dfrac{550+472}{4} =$ $= 255.5$ mm, h = 35 mm).

For a square plate with free edges, a side equal to r = 488.4 mm and h = 35 mm (flange No. 2) and waveforms with two nodal lines passing through the midpoints of the sides α = 14.1, respectively, the oscillation frequency is $f_{\text{square plate}} = 526$ Hz ($f_{\text{square plate}} = 480$ Hz for the flange No. 1 at r = 511 mm and h = 35 mm).

Oscillations of the body can be considered as oscillations of a cylindrical shell. The frequencies of natural radial vibrations of the shell for the case of one half-wave n = 1 along its length can be calculated by the formula V.E. Breslavsky [13]

$$f_{\text{radial vib. } i} = \frac{1}{2\pi}\sqrt{\frac{B}{mr}\,\frac{\left(1-\mu^2\right)\lambda^2 + k\left(\lambda^2+i^2\right)^4}{\lambda^4 + i^2\left(1+2\lambda^2\right)+i^4}}, \quad i = 2, 3, \ldots, \tag{6}$$

$$B = \frac{E\delta}{1-\mu^2};\ \lambda = \frac{\pi r}{l};\ k = \frac{\delta^2}{12 r^2},$$

where i is the number of nodal lines parallel to the generatrix; $\delta = (D_0 - d)/2 = 17{,}5$ mm is the shell thickness; D_0 is the outer diameter of the shell; $m = \rho\delta$ is the mass of the shell element; $r = (D_0 + d)/4 + \delta/2 = 224$ mm is the radius of the shell; $l = 540$ mm is the length of the shell.

The lowest frequency of radial vibrations of the shell (i = 2, n = 1) is $f_{\text{radial vib}}$ = 1147 Hz.

The calculation results by formulas (3)–(6) are summarized in column 5 of the Table 1. The obtained frequencies are located in those rows of the Tab. 1 for which the waveform shown in Fig. 2–8, is closest to the form of vibrations of the element in question.

As follows from the table, the frequencies of the body's own vibrations obtained by the FEM "corresponding" to the forms of bending vibrations in the plane of the ring quite well coincide with analytical solutions. This can be explained as follows. The presence of a cylindrical shell attached to the flange can be taken into account by increasing the thickness h of the ring. In accordance with (3), this does not lead to a change in the oscillation frequency, since the relation J_z/S, will remain unchanged.

The natural frequencies of the body, obtained by the FEM, "corresponding" to the forms of bending vibrations from the plane of the ring significantly exceed the results of the analytical solution (4), especially on the first forms of vibration. This can be explained by the fact that the cylindrical part of the housing for the flange acts as a boundary condition, preventing the movement of the sections of the flange in the direction perpendicular to its plane. In those cases when these boundary conditions play a more important role in determining the proper form of oscillations, the differences are more significant.

Similar considerations can be made when comparing the frequencies of the natural vibrations of the body obtained by the FEM and the natural frequencies of the plates and the cylindrical shell.

Let us analyze the dependence of the frequencies obtained according to (3)–(6) on the parameters of the housing design. To do this, we rewrite formulas (3)–(6) in a form that reveals the dependence of the obtained frequency value on the case parameters. These are body materials, values describing flanges: external (D) and internal (d) diameters, flange thicknesses (h), and quantities describing a cylindrical shell: external (D) and internal (d) diameters, body length (l). Formulas (3)–(6), respectively, will take the form

$$f_{\text{bend1}\,i} = \frac{2}{\sqrt{3}\pi}\sqrt{\frac{E}{\rho}}\sqrt{\frac{i^2\left(1-i^2\right)^2}{1+i^2}}\frac{(D-d)}{(D+d)^2}, \quad i = 2, 3, \ \ldots; \tag{7}$$

$$f_{\text{bend2}\, i} = \frac{4\sqrt{\frac{2}{3}}}{\pi}\sqrt{\frac{E}{\rho}}\sqrt{\frac{i^2\left(i^2-1\right)^2}{i^2+1+\mu}}\frac{h}{(D+d)^2},\ i=2,3,\ \ldots; \tag{8}$$

$$f_{\text{plate}} = \frac{2h\alpha}{\pi(D+d)^2}\sqrt{\frac{E}{\rho}\frac{1}{12\left(1-\mu^2\right)}}; \tag{9}$$

$$f_{\text{radial vib.}\, i} = \frac{1}{2\pi}\sqrt{\frac{E}{\rho}\frac{1}{\frac{D_0+d}{4}\left(1-\mu^2\right)}\frac{\left(1-\mu^2\right)\left(\pi\frac{D_0+d}{4l}\right)^2+\frac{\left(\pi\left(D_0-d\right)\right)^2}{12}\left(\left(\pi\frac{D_0+d}{4l}\right)^2+i^2\right)^4}{\left(\pi\frac{D_0+d}{4l}\right)^4+i^2\left(1+2\left(\pi\frac{D_0+d}{4l}\right)^2\right)+i^4}},$$

$$i=2,3,\ \ldots \tag{10}$$

From the obtained formulas it follows that all frequencies depend on the parameter $\sqrt{E/\rho}$, determined by the material used. Obviously, replacing aluminum alloy with steel will lower frequencies by about 3 %.

In Fig. 9 shows graphs illustrating the dependence of the frequency values on the selected geometric parameter of the case (other parameters are considered unchanged). Parameter change is set as a percentage of the nominal value.

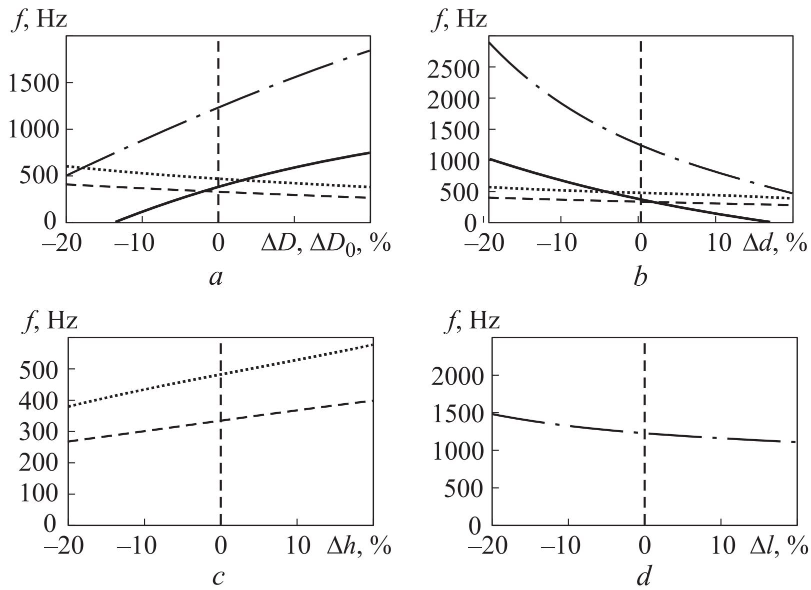

Fig. 9. Graphs of the dependence of frequencies on changes in the geometric parameter of the case (—— is f_{bend1}; – – – is f_{bend2}; ········ is f_{plate}; – · – is $f_{\text{radial vib}}$)

We define the approximate sensitivity coefficients of the vibration frequency values to a particular parameter of the housing structure as a partial derivative of the corresponding formula for this parameter multiplied by the parameter value. The calculation results for the natural frequencies calculated by formulas (3)–(6) from Table 1 are given in Table 2. Sensitivity factors for rings and plates, for example, were determined for flange No. 1

Table 2

Approximate sensitivity coefficients

Type of vibrations	Parameter	Approximate sensitivity coefficient
Bending vibrations in the plane of the ring (3)	$\frac{D-d}{2}$	9.86
	h	0
	r	3
Bending vibrations perpendicular to the plane of the ring (4)	$\frac{D-d}{2}$	0
	h	9.5
	R	–2.6
Vibrations of planes (5)	h	13.7
	r	–1.9
Vibrations of a cylindrical shell (6)	δ	44.1
	l	–2.8
	r	0.8

From Fig. 9 and Table 2 it follows that:

– a change in the value of the inner (*b*) and outer (*a*) diameter of the rings to a greater extent affects the frequency of bending vibrations in the plane of the ring and the frequency of radial vibrations of the cylindrical shell;

– frequencies of bending vibrations perpendicular to the plane of the ring and plate vibrations linearly depend on the thickness (*c*) of the flanges;

– the length of the radial vibrations of the cylindrical shell is noticeably affected by its length (*d*).

It should be noticed that the frequencies of elastic bodies determined by dependences (1)–(6) also depend on parameter i. That is, a change in one size of a housing element will lead to a change in several frequencies of natural vibrations.

Experimental studies of the frequencies of natural vibrations of the body. For experimental determination of natural frequencies, the body was hung on a flexible suspension. The case was loaded by mechanical shock using a pendulum installation. P_x, P_z are points of application of the load (the index indicates the direction of the force relative to the axes of the housing). The load application points are shown in Fig. 10.

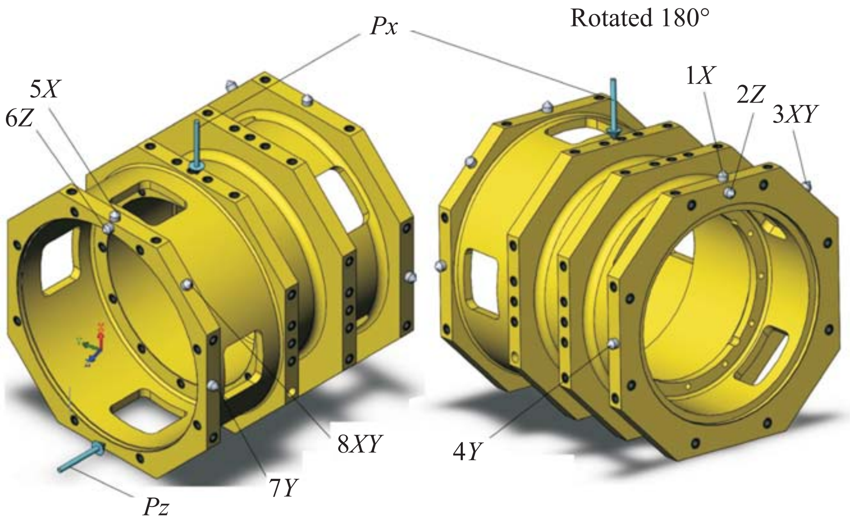

Fig. 10. Places of application of the load and installation of piezoaccelerometers

For registration of accelerations, 8 AP11 piezoelectric accelerometers (conversion coefficient (11 ± 4) pCl/g, amplitude of recorded accelerations up to 5000 g, operating frequency range 2–15 000 Hz) on VGO-1 (ВГО-1) adhesive sealant were installed on the case. In Fig. 10 shows the installation locations of piezo-accelerometers with the symbols 1*X*, 2*Z*, 3*XY*, 4*Y* are on flange No. 2 and 5*X*, 6*Z*, 7*Y*, 8*XY* are on flange No. 1 (the letter indicates the direction of the sensitivity axis of the piezoaccelerometer relative to the axes of the housing).

To register the signal from piezoelectric accelerometers, a signal amplifier was used (the nominal limit of charge conversion $Qv = 10 \cdot 10^4$ pCl, the nominal conversion coefficients K are from 0.01 mV/pCl to 10 V/pCl, the upper cutoff frequency is 10 kHz) and the ADC RSRC 2004 (measurement limits of the input voltage ± 1, ± 5 V, the maximum sampling frequency per channel 125 kHz, the number of bits of the ADC-14).

As a result of the tests, four loads were carried out: two along the X axis (experiments No. 1, 2) and two along the Z axis (experiment No. 3, 4). The signals from piezoelectric accelerometers were recorded with an upper cutoff frequency $f_{\text{up}} = 10$ kHz. After registration, in order to eliminate the industrial frequency of 50 Hz, the signals were filtered with a lower cutoff frequency $f_{\text{low dig}} = 50$ Hz with a fourth-order Butterworth digital filter.

A typical shape of the signal recorded by a piezoaccelerometer is shown in Fig. 11. The spectral density of this signal is shown in Fig. 12. The signals of piezoelectric accelerometers have a duration of about 50 ms, which provides a frequency step of 6–15 Hz.

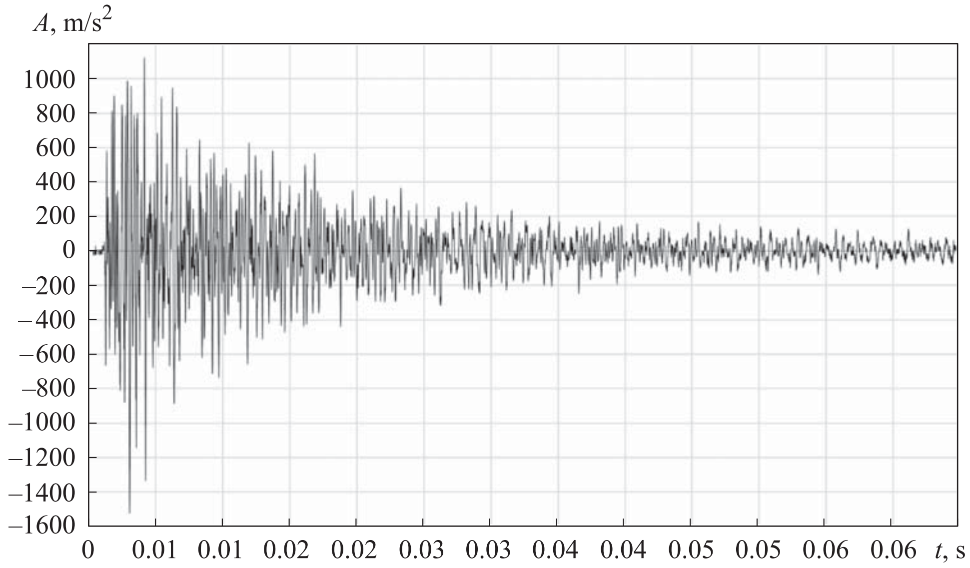

Fig. 11. Schedule of recorded shock acceleration with a 8*XY* piezoaccelerometer in experiment No. 2 without digital filtering

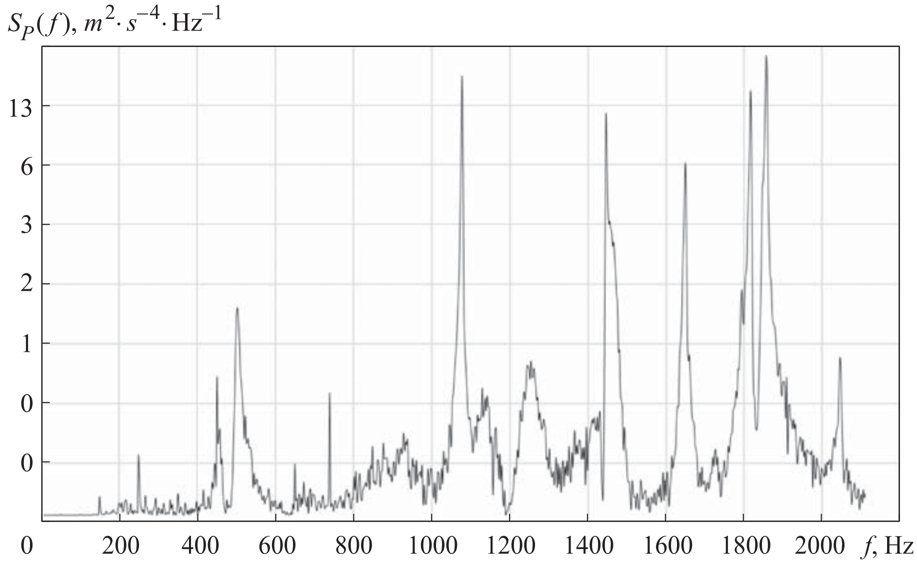

Fig. 12. Spectral density (S_P) graph of the recorded shock acceleration

The values of the natural frequencies were determined from the spectral density graphs of the signals obtained using the fast Fourier transform (FFT) as local extrema [15]. The results of determining the natural frequencies of the case are given in Table 3.

Table 3

The values of the oscillation frequencies determined from the peaks of the spectral graphs the density of the signals recorded by piezoelectric accelerometers

Flange No. 1								Flange No. 2							
5*X*		6*Z*		7*Y*		8*XY*		1*X*		2*Z*		3*XY*		4*Y*	
459	**462**	459	462	**459**	**462**	**502**	**504**	459	454	459	460	500	450	459	462
459	**459**	450	**459**	454	**459**	450	501	450	459	450	459	450	500	450	**459**
790	800	790	801	801	800	801	740	**801**	**801**	801		**875**	**876**	**801**	**801**
		800	794	739	794	739		**801**	794			740	**878**	**801**	**794**
1078	**1074**	**1078**	**1074**	**1078**	**1074**	**1078**	**1074**				1080				
1080	**1073**	**1080**	**1073**	**1080**	**1073**	**1080**	**1073**				1073				

In the Table 3 the following notation is accepted: frequencies obtained in different experiments are located in a square according to the diagram

experiment1(P_x)	experiment2(P_x)
experiment3(P_z)	experiment4(P_z)

. Bold letters indicate the frequencies whose peak amplitude corresponds to the condition $A > 0.1A_{\max}$, where $A_{\max}$ is the maximum peak amplitude.

The italics indicate the frequencies the amplitude of the peak at which meets the condition $A < 0.1A_{\max}$.

The frequency ranges defined in Table 3 are also given in Table 1 (column 6) in the corresponding rows.

To assess the compliance of the experimental and obtained by the FEM calculation, we will compare them using the formula

$$\Delta = \frac{f_E - f_C}{f_E} \cdot 100\,\%,$$

where f_C is natural frequencies of the case obtained using the FEM; f_E is the average value of the peaks of the spectrum.

The calculation results are given in Table 4.

Table 4

Comparison of experimental and calculated data

No.	f_C, Hz	f_E, Hz	Δ, %
1	463.93	464	0.01
2	505.58	464	8.96
3	843.78	798	5.73
4	908.77	798	13.88
5	1016	1076	5.57
6	1074.3	1076	0.15
7	1074.4	1076	0.14
8	1081.5	1076	0.51

From the Table 4, fairly good convergence of the experimental and calculated data follows.

Analyzing the obtained experimental data, the following can be noted. Considering that piezoaccelerometers well record vibration modes at which the cross-sectional displacement coincides with the sensitivity axis of piezoelectric accelerometers, 1*X*, 3*XY*, 4*Y*, 5*X*, 7*Y*, 8*XY* piezoaccelerometers well record vibrations “corresponding” to bending vibrations in the plane of the ring (frequencies 450–501, 1073–1080 Hz for flange No. 1; frequencies 740–878 Hz for flange No. 2), since in this case the displacements of the sections are radial in nature and coincide with the sensitivity axes of these sensors.

In other cases, when the sensitivity axis of the piezoelectric accelerometer is perpendicular to the displacement of the cross sections for a certain waveform, no peaks were noted on the graphs of the spectral density of the acceleration signal at a frequency corresponding to this waveform.

Experimental studies confirm the results of FEM calculations.

Conclusion. Using the FEM to determine the natural frequencies of oscillations of complex structures allows you to get the most accurate solutions, although it requires significant computational resources. The use of analytical solutions when replacing elements of a real structure with simpler elements allows one to find those design parameters that determine the values of the corresponding natural frequencies of the structure’s vibrations, as well as assess the effect of changes in these parameters on the value of the natural frequency of vibrations. This will help in the design of the structure, the reaction of which should have a given spectral composition.

When experimentally determining the natural frequencies of oscillations using piezoaccelerometers, in order to increase the accuracy of the results obtained, it is necessary to strive for the sensitivity axes of the used sensors to coincide with the displacements of the structural elements on the studied modes of vibration, and also to ensure that the installed piezoaccelerometers do not fall into the nodal points of their own waveforms.

Translated by K. Zykova

REFERENCES

[1] Yaushev A.A., Taranenko P.A., Zhestkov A.V., et al. Computational and experimental study of frequencies and natural mode of welded shell of coriolis flowmeter. *Vestnik YuUrGU. Seriya "Matematika. Mekhanika. Fizika"* [Bulletin of the South Ural State University, series "Mathematics. Mechanics. Physics"], 2018, vol. 10, no. 1, pp. 45–51 (in Russ.). DOI: 10.14529/mmph180106

[2] Ponomarev I.S., Makhnovich S.V., Pantileev A.S. Peculiarities of experimental determination of frequencies and shapes of eigenmodes of cylindrical shells. *Nauchnyy vestnik NGTU* [Science Bulletin of the NSTU], 2016, vol. 64, no. 3, pp. 44–58 (in Russ.). DOI: 10.17212/1814-1196-2016-3-44-58

[3] Ryabov A.V., Katanosov A.E., Trubaev A.I., et al. Computational and experimental investigations of the dynamic characteristics of the blades of the model Kaplan turbine. *Problemy mashinostroeniya* [Journal of Mechanical Engineering], 2014, vol. 17, no. 1, pp. 21–26 (in Russ.).

[4] Mezhin V.S., Obukhov V.V. The practice of using modal tests to verify finite element models of rocket and space hardware. *Kosmicheskaya tekhnika i tekhnologiya* [Space Technique and Technologies], 2014, no. 1, pp. 86–91 (in Russ.).

[5] Proskurin A.V. Vosproizvedenie udarnykh uskoreniy v laboratornykh usloviyakh [Laboratory reproduction of shock accelerations]. Snezhinsk, RFYaTs-VNIITF Publ., 2017.

[6] Gokhfel'd D.A., Getsov L.B., Kononov K.M., et al. Mekhanicheskie svoystva staley i splavov pri nestatsionarnom nagruzhenii [Mechanical properties of steels and alloys under non-stationary loading]. Ekaterinburg, UrO RAS Publ., 1996.

[7] PNAE G-7-002–86. Normy rascheta na prochnost' oborudovaniya i truboprovodov atomnykh energeticheskikh ustanovok. Gosatomenergonadzor SSSR [Norms for stress calculation of the equipment and pipelines for atomic power stations. USSR State Atomic Power Supervision]. Moscow, Energoizdat Publ., 1989.

[8] Pisarenko G.S., Yakovlev A.P., Matveev V.V. Spravochnik po soprotivleniyu materialov [Handbook on strength of materials]. Kiev, Naukova dumka Publ., 1975.

[9] Zakharov Z.P., red. Fiziko-mekhanicheskie svoystva konstruktsionnykh materialov i nekotorye sovremennye metody ikh issledovaniya [Physical-mechanical properties of constructional materials and some modern methods of their research]. Moscow, TsNIIatominform Publ., 1982.

[10] APM Structure3D. Rukovodstvo pol'zovatelya. Sistema rascheta i proektirovaniya detaley i konstruktsiy metodom konechnykh elementov. Versiya 16 [APM Structure 3D. User manual. System for calculation an engineering of parts and constructions using finite elements method. Version 16]. Korolev, NTTs Avtomatizirovannoe Proektirovanie Mashin Publ., 2018.

[11] Timoshenko S.P., Young D.H., Weaver W. Vibration problems in engineering. Van Nost. Reinhold, 1955.

[12] Filippov A.P. Kolebaniya mekhanicheskikh system [Oscillations of mechanical systems]. Kiev, Naukova dumka Publ., 1965.

[13] Skubachevskiy G.S. Aviatsionnye gazoturbinnye dvigateli. Konstruktsiya i raschet detaley [Aviation gas-turbine engines. Parts construction and calculation]. Moscow, Mashinostroenie Publ., 1969.

[14] Biderman V.L. Teoriya mekhanicheskikh kolebaniy [Theory of mechanical oscillations]. Moscow, Vysshaya shkola Publ., 1980.

[15] Smith S.W. The scientist & engineer's guide to digital signal processing. California Technical Publ., 1997.

Proskurin A.V. — Dr. Sc. (Eng.), Deputy Chief Design Officer, Russian Federal Nuclear Center — Zababakhin All-Russian Scientific Research Institute of Technical Physics (p/o box 245, Vasileva ul. 13, Snezhinsk, Chelyabinsk Region, 456770 Russian Federation).

Shlishevskiy A.V. — Head of Group, Russian Federal Nuclear Center — Zababakhin All-Russian Scientific Research Institute of Technical Physics (p/o box 245, Vasileva ul. 13, Snezhinsk, Chelyabinsk Region, 456770 Russian Federation).

Please cite this article as:

Proskurin A.V., Shlishevskiy A.V. Calculated-experimental determination of values natural frequencies oscillations of the case and parameters of the case determining these values. *Herald of the Bauman Moscow State Technical University, Series Mechanical Engineering*, 2019, no. 6, pp. 89–104. DOI: 10.18698/0236-3941-2019-6-89-104