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ENDOCHRONIC THEORY OF VISCOPLASTICITY. AN EXAMPLE OF ITS PRACTICAL IMPLEMENTATION FOR HIGHLY FILLED POLYMERIC MATERIAL

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Abstract

Complex mechanical behaviour of materials differs from elastic deformation and implies plasticity, creep, changes in Poisson's ratio during deformation and other microscopic phenomena. A detailed description of them can lead to the most complete and accurate equations, but currently it is impossible in practice. To circumvent these difficulties we usually use a phenomenological approach. In this case, a mathematical model describing the experimental data for the material with the required degree of accuracy is created. In 1971, K. Valanis introduced the term "endochronic" to the theory of plasticity. The article is prepared using the materials of lectures given by the author. The paper is for readers who are not proficient in the endochronic theory framework. The article is prepared using the materials of lectures given by the article presents the results of using the endochronic theory to describe complex mechanical behavior of a highly filled polymer material (HFPM). We determined the internal (endochronic) time function based on test results concerning tension, compression, shear and shear combined with axial compression. We then used this function in the finite element method (FEM) to solve the problem of a rigid die indenting a HFPM volume. We show the advantage of using the endochronic theory in the FEM

Keywords

Plasticity, viscoplasticity, thermodynamics, internal variables, K. Valanis, endochronic theory, highly filled polymer material, finite element method, testing of materials

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Nonlinear behavior, plasticity, creep, the different behavior at loading and unloading, changes in Poisson's ratio of the material due to processes taking place on the micro level. A detailed description of them can lead to the most complete and accurate equations, but currently it is practically impossible.

To circumvent these difficulties usually use a phenomenological approach. In this case, a mathematical model describing the experimental data for the material with the required degree of accuracy is created. This approach, which can be called an approach at the macro level, led to the creation of a large number of models: rigid-plastic body, elastic-plastic body, visco-elastic models Kelvin and Maxwell etc.

To bring together micro- and macroapproach to describe the mechanical behavior of materials used the theory based on the method of non-equilibrium thermodynamics with internal variables [1–6]. Internal variables describe the micro-processes at the macro level. These theories develop on the basis of the ideas expressed by B. Coleman, K. Truesdell, L.I. Sedov. Such theories propose a more general mathematical model of the mechanical behavior of the material, have a more reliable physical basis than the purely phenomenological theories. In the same time they are quite convenient and simple for use.

The number of hidden (internal) variables independent from a specific process. It depends on the material properties and the accuracy of the description of the material behavior. The internal variables can be either scalar or tensor.

In the general case, the choice of internal coordinates is not an easy task. The experimenter can produce certain effects and to measure the reaction of the material to them. Material from the point of view of continuum mechanics is a "black box", the internals of which we are interested only to the extent that it determines reaction to the external impact. The optimal choice of comfortable internal parameters, the minimum number, but with enough information about the reaction of the material is not a trivial task. The laws of thermodynamics give you only a general direction to find these internal variables.

The theory of viscoplasticity with internal parameters was proposed by K. Valenis [1] in the most convenient way for practical use. He proposed to use instead of real time t, measured by clock, a dummy internal time z, which dependent of material properties and history of deformation. He called the internal time z "endochronic". Therefore, the theory using the parameter z called "endochronic". It has been successfully used to describe the mechanical behavior of metals, concrete, soil, highly filled polymer materials, composite materials. One of the modern reviews on the application of endochronic theory is given in [7].

The endohronic theory uses the assumption that the present state of the material depends on all the occurred processes of deformation, but does not depend on the sequence in which they followed each other. In other words all the history of deformation that led to the same strain are equivalent In other words all the history of deformation that led to the same strain are equivalent.

This variable z is used to describe the processes of deformation, accompanied by microstructural changes. Internal time monotonically increases with the growth of some convenient measure of deformation — the scale of internal time θ.

To satisfy the second law of thermodynamics dz and $d\theta$ must always be nonnegative. In addition, z and θ must be a monotonic function of the deformation, or may be a case where two different levels of strain corresponds to the same value of θ.

The need for the introduction of endochronic time is due to the fact that during the deformation the internal parameters cannot be measure directly, but researchers can measure the components of strain ε_{ij} and temperature T at any point in any time t. Any increment dt corresponds to the increment of $d\varepsilon_{ij}$ and dT. The three increments will correspond to the increment of the internal time dz dependent of material properties.

In this work one of the options of endochronic theory is used to describe the mechanical behavior of highly filled polymeric material (HFPM). Unconventional properties of such a material is described in the article [8].

Consider the specific function of the internal time for materials of the type HFPM.

The HFPM is isotropic material. Increment internal time should not be sensitive to the coordinate transformation and hence must be expressed through the invariants of the tensor of the increments of deformation.

Since the deformation is assumed to be small, the third invariant can be neglected.The increment of the internal time should not depend on the first invariant of the tensor of the increments of the deformations, as the volumetric strain is purely elastic.

The differential forms of the endochronic theory equations are obtained in [9] and it is shown that the mechanical model for this theory is a set of Maxwell elements, but working not in real time but in endochronic time. Also in [9] it was shown that the theory of Kadashevich — Novozhilov (1958) can be obtained as a special case from the endochronic theory (1971).

To describe the behavior of HFPM, whose volumetric deformations are elastic, as a rule, one internal variable is sufficient. In this case, the equation linking stress and strain is written as [9]

$$ds_{ij} + s_{ij}dz = 2Gde_{ij}; \tag{1}$$

$$d\sigma^0 = 3K\varepsilon^0, \tag{2}$$

where s_{ij} — components of the stress deviator; e_{ij} — components of the strain deviator; σ^0 — volume stress; ε^0 — volume deformation; G — shear modulus; K — volume modulus, $K = E_0/(1-2\mu_0)$; E_0 — initial value of the modulus of elasticity; μ_0 — initial value of the Poisson ratio.

In matrix form for these relations take the form of

$$\{d\sigma\} + \{d\sigma'\} = [D]\ \{d\varepsilon\}, \tag{3}$$

where $\{d\sigma\}$ and $\{d\varepsilon\}$ — the vectors of increments of stress and strain; $\{d\sigma'\}$ is a vector of increments of additional stresses determined by the increment of the internal time; $[D]$ — a matrix of material properties.

The increment of internal time dz is determined by the formula

$$Dz = F(\sigma_{ij},\ \varepsilon_{ij})\,dz, \tag{4}$$

where $dz = \sqrt{de_{ij}\,de_{ij}}$.

The second term in equation (1) divided by $2G$ is the increment of inelastic energy as part of the total energy. This imposes constraints of the kinematic nature on the function F. At $F = 0$ material deformed elastic. Under inelastic deformation $F = \sqrt{\dfrac{2G^2}{J_2(\sigma)}}$, where $J_2(\sigma)$ is the second invariant of the stress deviator.

The type of the internal time function and its coefficients was selected from the condition of the best description of the test results of samples made of HFPM [8].

The difference between experimental and theoretical curves of work [8] does not exceed 3 % in compression, 5 % in tension, 11 % in shear and shear with compression. At the same time, it was possible to describe different material behavior under tension and compression (different modules except for the initial point), different nature of change of Poisson ratio (increases under compression and decreases under tension), increase of shear modulus under application of compressive load, presence of hysteresis loops under cyclic compression and cyclic shift, etc. For an isotropic material, the internal time function is expressed through invariants of stress and strain tensors and has the following form

$$F\left(\sigma_{ij},\ \varepsilon_{ij}\right)=f_1/f_2; \qquad (5)$$

$$f_1=c_1+c_2\left[1+c_3I_2\left(\sigma\right)\right]\sqrt{I_2\left(\varepsilon\right)}\Big/\left\{\left[1+\ c_4I_1\left(\sigma\right)\right]\ \left[1+\ c_5I_2\left(\sigma\right)\sqrt{I_2\left(\varepsilon\right)}\ \right]\left[1+\ c_6I_1\left(\sigma\right)\right]\right\};$$

$$f_2=\left[1+c_7/\left(1\ +c_8f_1\right)\right]\Big/\left\{1+\left[1+c_9I_1^2\left(\sigma\right)\right]\sqrt{I_2\left(\sigma\right)}\,c_{10}f_1^2/\left(f_1^2+c_{11}\right)J_2\left(\varepsilon\right),\right.$$

where $I_1\left(\sigma\right)$ and $I_2\left(\sigma\right)$ — the first and second invariants of the stress tensor; $J_2\left(\varepsilon\right)$ — the second invariant of the strain deviator. The following coefficients are used for the described material:

$$E_0=43\text{ MPa};\ \mu_0=0,3;\ c_1=0,72;\ c_2=510;\ \ c_3=4,42\cdot10^5/E_0^2;$$
$$c_4=2060/E_0;c_5=354/E_0^2;\ c_6=66,9/E_0;\ c_7=496;\ \ c_8=\ 20,7;$$
$$c_9=73,9\cdot10^9/E_0^3;\ c_{10}=0,0023-0,63\cdot I_1\left(\sigma\right)/E_0;\ \ c_{11}=0,00152.$$

Quite a complex type of function of internal time and a large number of factors are a fee for a simple and convenient way to organize the computing process for the material with complex mechanical behavior.

The correctness of the choice of the internal time function check phenomenologically. When it is substituted in equation (1) and (2), these equations must accurately describe the experimental data on the material under consideration.

As an example, the problem of pressing a hard stamp into the elastic-plastic space with the properties of a real material HFPM was solved. In this case, each finite element is in a multi-axial stress state.

The diameter of the cylinder from HFPM is $2r_0$ = 35 mm, its height $2H$ = 70 mm, the diameter of the stamp $2R$ = 10 mm.

To obtain a solving system of equations by the finite element method (FEM), we use as usual the principle of the possible moving. The calculated body is divided into M finite elements (Fig. 1).

Using the traditional FEM procedure, we obtain a nonlinear system of equations. The difference this system with the system of equations of standard FEM is that there are additional nodal forces, defined by the increments of the internal time. This system of equations is solved by iteration. At each step of load increment the elastic problem is initially solved. In this case, $\left\{d\sigma'\right\}=0$. Then for each element the increment of internal time is calculated and the M — dimensional vector of these values is remembered (array DZ). Also, for each element by equation (1), (3) additional stresses are calculated on the basis of which the $k \times l$ — dimensional vector of additional forces is formed (k is the number of nodes, l is the number of stress components). This array is included in the determination of the vector of external loads in the subsequent iterations.

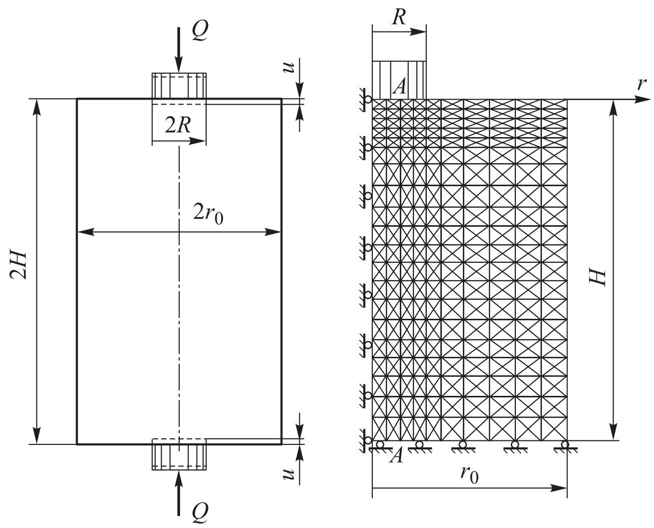

Fig. 1. The scheme of division into finite elements

The solution process for a given value of the load continues until the arrays DZ on the previous dz_i and subsequent dz_{i+1} iterations will not coincide with the accuracy 0,001 dz_{i+1}.

The total force acting on the stamp indicated by Q. To the value Q = 8,58 kN loading is carried out in five stages.

The results of calculations are shown in Fig. 2 and 3. Fig. 2 show the dependence of the depth of penetration of the stamp u from the magnitude of the force Q. The solid line corresponds to the solution of FEM, where the endochronic theory is used to describe the plastic property, the bar–dotted line is the analytical J. Boussinesq solution for semi-infinite space.

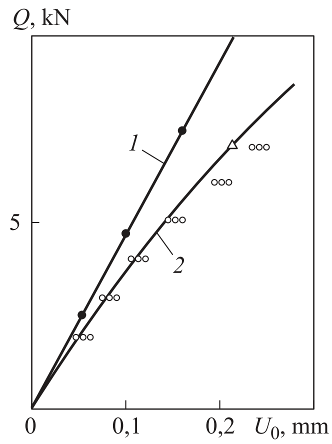

Fig. 2. The movement u_0 of the stamp depending on the magnitude of the force Q:

1 — Boussinesq solution; *2* — the solution of FEM, where the endochronic theory is used to describe the plastic property of HFPM

The accuracy of the description of the HFPM behavior under multi-axial loading and the possibility of the proposed internal time function were tested by experiment. The scheme of the experiment corresponds to the calculated scheme of the FEM of the Fig. 1. Mutual approach of the stamps was determined by the readings of the three dial indicators with division 0.001 mm installed with equal pitch in the circumferential. Five samples were used.

The experimental points are put on the picture Fig. 2.

The first three tests were carried out with a step increase in the load *Q* with taking of indications at each step. Their results are shown by circles. The difference between theory and experiment in this case increases with the growth of force *Q*, but does not exceed 17 %. This is because HFPM manifests properties of creep and during the test it accumulated creep deformation. The relations (3), (4) the process of creep is not considered.

The second series of experiments was carried out at a higher loading speed. Load up to $Q = 7$ kN was created continuously, without taking readings in the intermediate stages. These results are shown by triangles. The difference between theoretical and experimental results does not exceed 3…5 %. The rate of deformation in the second series of experiments was almost the same as in tests, the results of which determine the parameters of the internal time function.

Fig. 3 shows the distribution of axial ε_z and tangential ε_θ deformation under the stamp in the nearest to the axis *z* of the column of finite elements. The dashed lines correspond to the elastic solution, the solid line represents the solution using endochronic theory. Curves *1* are obtained at a load of $Q = 3{,}43$ kN, curves *2* are a load of $Q = 8{,}58$ kN.

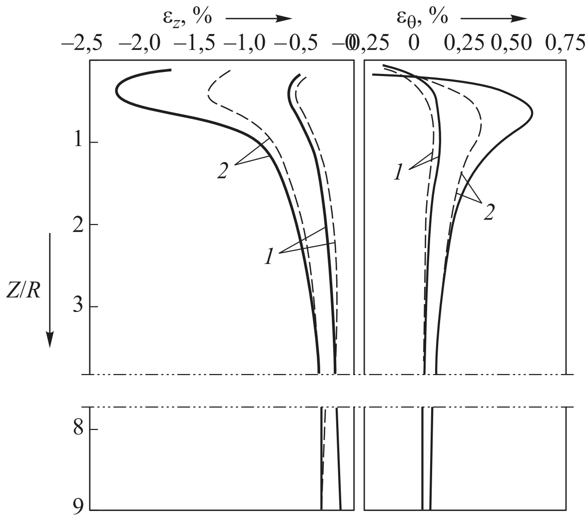

Fig. 3. The distribution of axial ε_z and tangential ε_θ deformation under the stamp in the nearest to the axis *z* the column of finite elements:

1 — curves are obtained at a load of $Q = 3{,}43$ kN; *2* — curves are a load of $Q = 8{,}58$ kN

The results show the advantage of the endochronic theory for calculating the stress-strain state under low-cycle loading. A computational program based on endochronic theory developed only for active loading is transformed into a program for low-cycle loads by adding only one operator, which determine the changes in the sign of load increment.

Conclusion. Based on the results obtained, it can be concluded that the theory of visco-plasticity using the internal time parameter is simply built into the FEM in terms of simplifying program debugging and saving computational time. After defining a rather complicated function of the internal time according to the results of simple experiments for further calculation of structures does not cause significant difficulty.

Particularly great advantage is given by endocrine theory in solving problems of low-cycle loading.

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