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PNEUMATIC DAMPER MULTICRITERIA OPTIMIZATION IN THE FREE-PISTON STIRLING ENGINE DISPLACER

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Abstract

The free-piston Stirling engine is a complex self-oscillating system, which parameters should be strictly coordinated with each other to ensure the unit operability. One of such coordination tasks is to ensure the balance of energy input to and output from the engine displacer. In this case, energy output depends on the dissipative forces, and energy output is provided by the displacer rod and depends on its diameter. The need to select rod diameter complicates and slows down the engine design. In addition, the system possible instability could lead to the engine racing. To solve this problem, it is proposed to install a pneumatic damper on the displacer rod ensuring the dissipative forces growth with an increase in the displacer amplitude above its nominal value. Thermodynamic model was created to evaluate the damper characteristics and select its parameters. MATLAB software package was used as the development environment. The system of differential equations was integrated using the Runge — Kutta family method. The damper multicriteria optimization problem was formalized, and two objective functions with five independent parameters were compiled. Optimization was performed using the genetic algorithm, and the Pareto front was built. To evaluate results of the work performed, the damper mathematical model was integrated into the Stirling engine mathematical model, and the working process was calculated with selected damper parameters. It is demonstrated that the developed pneumatic damper prevents an increase in the displacer oscillation amplitude ensuring system stability and reducing time for the unit design

Keywords

Stirling engine, Beale engine, free-piston engines, mathematical model, thermodynamic model, pneumatic damper, multicriteria optimization

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Introduction. Ecological trends and world politics each year are increasingly directing the power engineering development towards alternative and renewable sources of electrical energy. Development of solar energy, cogeneration plants, heat recovery systems and many other areas make it possible to test both new technological and previously known solutions from the alternative side.

At present, Stirling engine could be recognized as one of the promising areas in the alternative energy development. Power plants built on its basis have a number of important advantages, which include fairly high efficiency (up to 39 %) [1, 2], long autonomous operation term (up to 120 000 hours) [3, 4] and ability to work using an arbitrary source of heat (including nuclear reactor, radioisotope energy source, solar radiation, etc.).

Due to its advantages, modern Stirling engines are used in various areas: solar energy, cogeneration plants [5], heat recovery systems [6], special [7, 8] and space technologies [9].

As a rule, free-piston version of the modern Stirling engine, also called the Beale engine, is being considered, its schematic diagram is shown in Fig. 1. Such an engine has no classic drive mechanism, and thermal energy is converted into

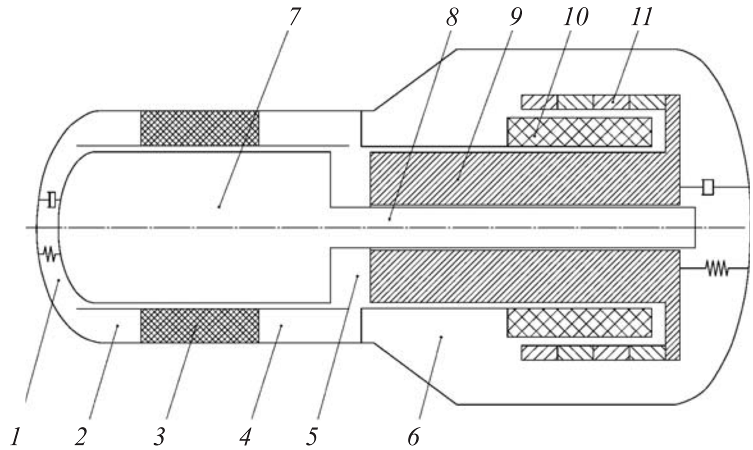

Fig. 1. Free-piston Stirling engine:
1 is expansion cavity; *2* is heating cavity; *3* is regenerator; *4* is cooling cavity; *5* is compression cavity; *6* is buffer cavity; *7* is displacer; *8* is displacer rod; *9* is working piston; *10* is alternator winding; *11* is alternator magnets

electrical energy due to linear alternator [10]. Absence of a drive mechanism makes it possible to significantly reduce friction losses increasing the engine efficiency, as well as to abandon the liquid lubrication system and thus increase its autonomous operation term. At the same time, this leads to the need to consider the free-piston Stirling engine as a complex self-oscillating system. One of disadvantages of such a system is the difficulty in achieving energy balance on the displacer, since energy supply to it is associated with the presence of a rod and depends on its diameter. Thus, this energy dissipates due to friction forces and presence of hydraulic resistance in the thermal circuit. In this case, with an increase in the displacer oscillations amplitude, energy supply to it also increases; and the dissipative forces growth could be insignificant. Disbalance could lead to both damping of self-oscillatory processes and engine racing, which will cause the displacer hitting the housing in a real engine or to incorrect calculation results in the case of mathematical simulation.

Ensuring energy balance on the displacer at the early computational and experimental stages leads to requirement for the engine fine-tuning, which requires significant time and labor.

Solution to this problem is not quite clear, and literature provides practically no description. Classical work [11] mentions that rubber annular shock absorption stops are installed on low-power engines in the rod end oscillation area. Articles [10, 12] describe the use of planar springs, and there is evidence of using a certain membrane placed between them. However, information provided is not sufficient to come to an unambiguous conclusion on its purpose, and the lack of any technical characteristics of this method does not allow it to be freely applied. There are also modifications of Stirling engines [10], where an electric alternator is installed on the displacer rod, and that could be used to control oscillations. However, such a technical solution obviously significantly increases complexity of the unit.

This work objective is to create a proprietary method ensuring energy balance on the displacer and making it possible to increase the operating cycle stability, as well as simplifying and accelerating the free-piston Stirling engine design and development.

Materials and methods used in solving problems, assumptions made. To maintain values of the displacer oscillation amplitude at the necessary level, a damping element is required providing minimum resistance at the nominal oscillation amplitude and maximum resistance, when the oscillation amplitude goes beyond the permissible values. Pneumatic damper was developed as such an element, which is a hollow cylinder and forms a cylinder-piston pair together with the displacer rod. Besides, a hole is provided in the cylinder for the working fluid operation in the damper with the engine buffer cavity. During engine operation, effective flow area of this hole changes by partial or complete blocking the displacer by the rod, as a result the dissipative forces depend on the latter oscillation amplitude.

To ensure the most efficient operation of a pneumatic damper, it is necessary during its design and development to select optimal combination of the flow section and the damper parameters as a whole. However, if there are several quality indicators, the problem of finding an optimal combination may not have a single correct solution. Thus, finding the most efficient parameters for a pneumatic damper is a multicriteria optimization problem.

There are various methods for solving the multicriteria optimization problem, such as scalar ranking [13] and genetic algorithm methods [14–16]. The most appropriate is the NSGA-II method [17] due to its simplicity and efficiency. This method also refers to the so-called “archive algorithms”, which makes it possible to save the most optimal generation samples in the archive from which a new population is produced. After that, the best samples of the new population replace certain samples in the archive, etc. This algorithm makes it possible to save all the most optimal samples found in the search process.

Optimality of the obtained solutions was evaluated by constructing the Pareto fronts [18].

For computational analysis of the pneumatic damper, its thermodynamic model was developed. Calculation scheme of the pneumatic damper is presented in Fig. 2.

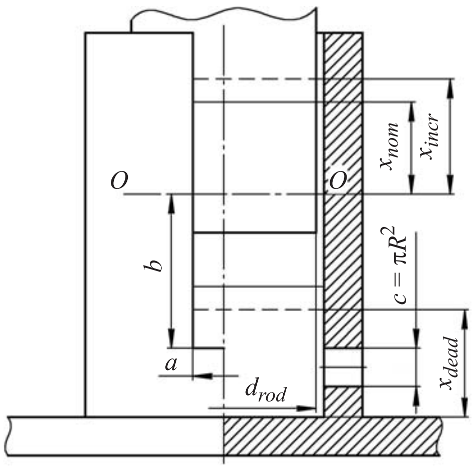

Fig. 2. Pneumatic damper calculation scheme:
a is hole width; b is hole length from the edge to the displacer rod oscillation axis; c is area of additional and always open section; d_{rod} is displacer rod diameter; x_{dead} is dead volume height; x_{nom} is displacer rod oscillation nominal amplitude; x_{incr} is displacer rod oscillation increased amplitude

When formulating the mathematical model for a pneumatic damper, a number of assumptions were made. The working fluid is ideal gas (helium). Heat exchange with the damper wall is missing. The sinusoidal law sets the displacer motion. Pressure in the buffer cavity is constant and equal to the average pressure of the Stirling engine. Mach number is limited to unity in order to exclude supersonic flows.

Mathematical model is described by two differential equations, i.e., the first law of thermodynamics for the dU open thermodynamic systems and the law of alteration in the dm working fluid mass in the damper. In this case, the G flow rate through the μF effective flow section is calculated using the $q(m)$ gas-dynamic function taking into account the Mach number.

The first law of thermodynamics for open thermodynamic systems is described by the following formula:

$$dU = -dA + dH,$$

where A is the work done by the working fluid inside the damper on the displacer rod; H is the working fluid flow enthalpy through the hole in the damper.

The law of mass alteration is described by the formula

$$dm = G\,dt.$$

Work done by the working fluid inside the damper on the displacer rod is calculated as:

$$A = p_{dempf}\, S_{rod}\, \frac{dx_{disp}}{dt},$$

where p_{dempf} is pressure in the damper; S_{rod} is the displacer rod area; x_{disp} is the displacer motion.

Enthalpy flow through the hole in the damper is calculated by the equation

$$\frac{dH}{dt} = c_p\, T_{dempf,b}\, G,$$

where c_p is the gas specific heat capacity at constant pressure; $T_{dempf,b}$ is the working fluid temperature in damper or in buffer cavity, respectively, depending on the flow direction.

Flow rate through the effective section is calculated as:

$$G = \frac{\pm\, \mu F P\, q(\mathrm{M}) \cdot 0.0161}{\sqrt{T_{dempf,b}}},$$

where μF is the damper hole effective flow section; $q(\mathrm{M})$ is the gas-dynamic function.

The gas-dynamic function could be represented as

$$q(\mathrm{M}) = \mathrm{M}\left(\frac{2 + (k-1)\mathrm{M}^2}{k+1}\right)^{\frac{0.5(k+1)}{1-k}},$$

where M is the Mach number determined from the pressure difference between the damper and the buffer cavity,

$$\mathrm{M} = \sqrt{\frac{2}{(k-1)}\left(\left(\frac{p_{in}}{p_{out}}\right)^{\frac{k-1}{k}} - 1\right)},$$

p_{in}/p_{out} is the pressure difference between the damper and the buffer cavity; k is the adiabatic index.

Flow section between the damper internal volume and the engine buffer cavity is conditionally divided into two terms. The first corresponds to the a width rectangular slot, which is partially or completely blocked by the rod during its motion. The second is a hole with the c constant area, which could be interpreted as a gap between the rod and the damper housing. Thus, effective cross section could be found as

$$\mu F = \max\left(a\left(x_{disp} + b\right),\ \ 0\right) + c.$$

The law of displacer motion is assumed to be sinusoidal and is described by the following equation:

$$x_{disp} = x_{nom,\ incr} \sin\left(2\pi\, ft\right),$$

where f is the frequency.

Mathematical model is realized in the MATLAB software package [19]. In integration, the standard MATLAB *ode45* solver for non-stiff systems of differential equations on the fourth order Runge — Kutta method is used.

Multicriteria optimization problem is formalized as follows. Two objective functions are compiled that return the average value per cycle power resistance of the pneumatic damper:

$$N_{resist} = \frac{\int\limits_{t_0}^{t_1} p_{dempf}\, S_{rod}\, u_{disp}\, dt}{t_{cycle}},$$

where t_0 is the working cycle start time; t_1 is the working cycle end time; t_{cycle} is the working cycle duration; u_{disp} is the displacer rate.

The first function should ensure the minimum resistance power condition at the nominal value of the displacer oscillation amplitude, the second — the maximum resistance power with an increase in the oscillation amplitude value of 3 mm:

$$-N_{resist}\left(x_{nom}, a, b, c, x_{dead}, d_{rod}\right) \to \max;$$

$$N_{resist}\ \left(x_{incr}, a, b, c, x_{dead}, d_{rod}\right) \to \min.$$

For independent parameters, the following ranges of values were chosen:

$$A = 0.0005\text{–}0.005\ \text{m};\ b = -\,0.03\text{–}0.03\ \text{m};\ c = 5\cdot 10^{-7}\text{–}1\cdot 10^{-5}\ \text{m}^2;$$

$$x_{dead} = 0\text{–}0.05\ \text{m};\ d_{rod} = 0.01\text{–}0.03\ \text{m}.$$

Multicriteria optimization problem was solved in the MATLAB software package using the built-in NSGA-II function with the following calculation parameters: population size is 200 points; number of generations is 100.

Results. Result of the genetic optimization are arrays of independent parameters and their corresponding objective function values for each population sample in all calculation generations. As the Pareto front (i.e., the set of Pareto optimal solutions to the optimization problem), non-dominated samples of the last hundredth generation population are taken.

Preliminary calculations demonstrated that the c additional hole area for optimal solutions tends to the lowest allowable range value. Therefore, this value was taken equal to $7.45 \cdot 10^{7}$ m^2 and was excluded from the list of independent variables for final optimization.

Final results of solving the optimization problem are shown in Figs. 3 and 4. Objective function values for all population values of all generations are presented in Fig. 3. Gray and light gray dots on the graph correspond to the algorithm earlier generations. Independent parameters distribution for Pareto optimal solutions of the last generation from the second objective function are provided in Fig. 4:

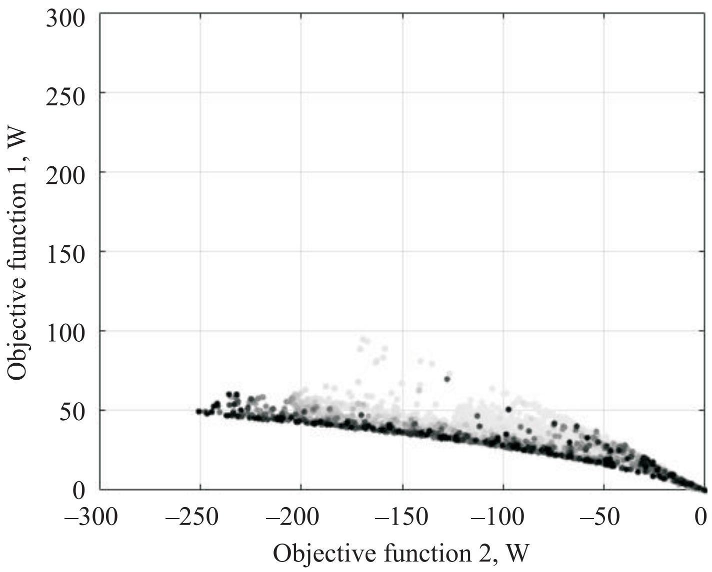

Fig. 3. Pareto front

– two sections could be distinguished in the Pareto front. The first section is characterized by low values of the dissipative power (to 25 W at the displacer nominal motion) and unstable parameter values, which is apparently due to their weak influence on the objective functions. The second section (more than 25 W) is of primary interest when choosing the damper dimension;

– the a hole width values on most of the Pareto front tend towards the 4 mm value;

– the b hole length tends to be equal to the x_{nom} displacer rod oscillation nominal amplitude at the top dead center. At the same time, the hole tends to be completely closed at the nominal value amplitude value and to open with growing oscillation amplitude to increase average pressure in the damper and, accordingly, the resistance power.

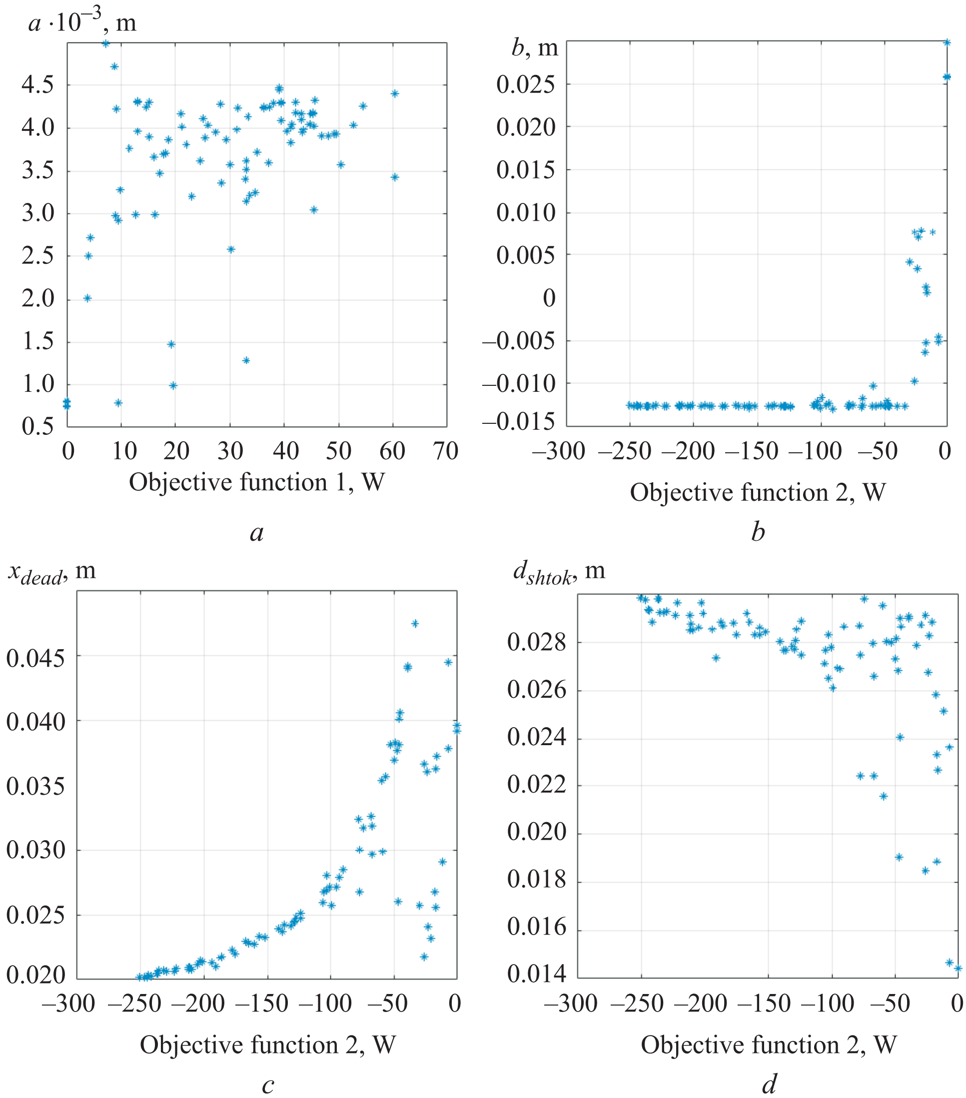

Fig. 4. Distribution of the calculated values of the hole width (a) and length (b), dead volume height (c) and rod diameter (d) according to the objective function (2)

The following conclusions could be made based on parameters distribution along the Pareto front:

– the x_{dead} dead volume height significantly affects the pneumatic damper resistance power value and will be chosen as the main parameter of its regulation;

– the d_{rod} displacer rod diameter also significantly affects the pneumatic damper resistance power value; however, power regulation by this parameter is not very convenient in practical terms, since this parameter also affects the main engine operation.

To evaluate results of the multicriteria optimization, it is necessary to integrate the pneumatic damper mathematical model into the free-piston Stirling engine mathematical model. Mathematical model [20] with a pre-configured FPS engine [12] was used for this purpose. Before that, the engine working process was preliminary calculated without a pneumatic damper in order to further evaluate work results. Calculation results are shown in the form of the displacer motion graph (Fig. 5, *a*) and the engine indicator diagram (Fig. 5, *b*) for two rod diame-

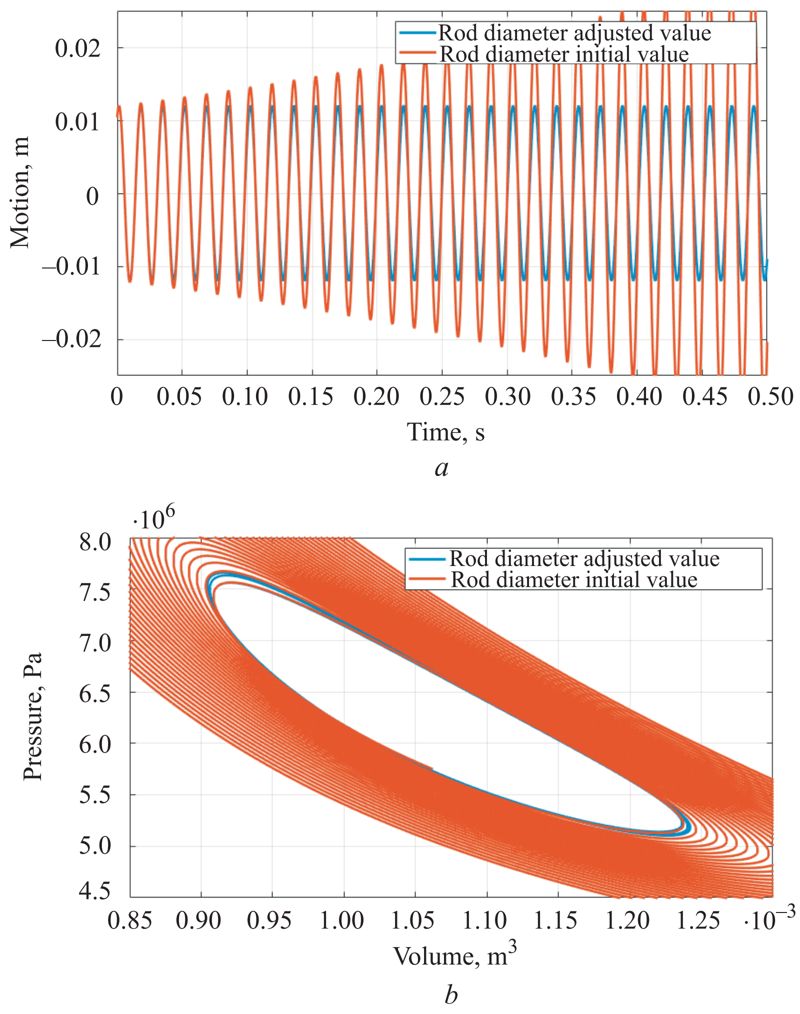

Fig. 5. Calculation results without pneumatic damper:
a is displacer motion; *b* is engine indicator diagram

ters: 20 mm (corresponds to the real FPS engine) and 15 mm (value adjusted to ensure power balance on the displacer). It should be noted that the system demonstrates high sensitivity to the rod diameter even without using the special measures.

It is advisable to compare engine operation with and without a damper at the same rod diameter (0.02 m). Thus, it was decided to exclude independent parameters from the list and reoptimize it. After that, a point belonging to the Pareto front was selected with the following pneumatic damper parameters: $a = 0.004$ m; $b = -0.013$ m; $c = 7.45 \cdot 10^{-7}$ m^2; $x_{dead} = 0.026$ m.

After selecting the pneumatic damper parameters, the engine working process was calculated. Calculation results are provided in the form of the displacer motion graph (Fig. 6, *a*) and the engine indicator diagram (Fig. 6, *b*).

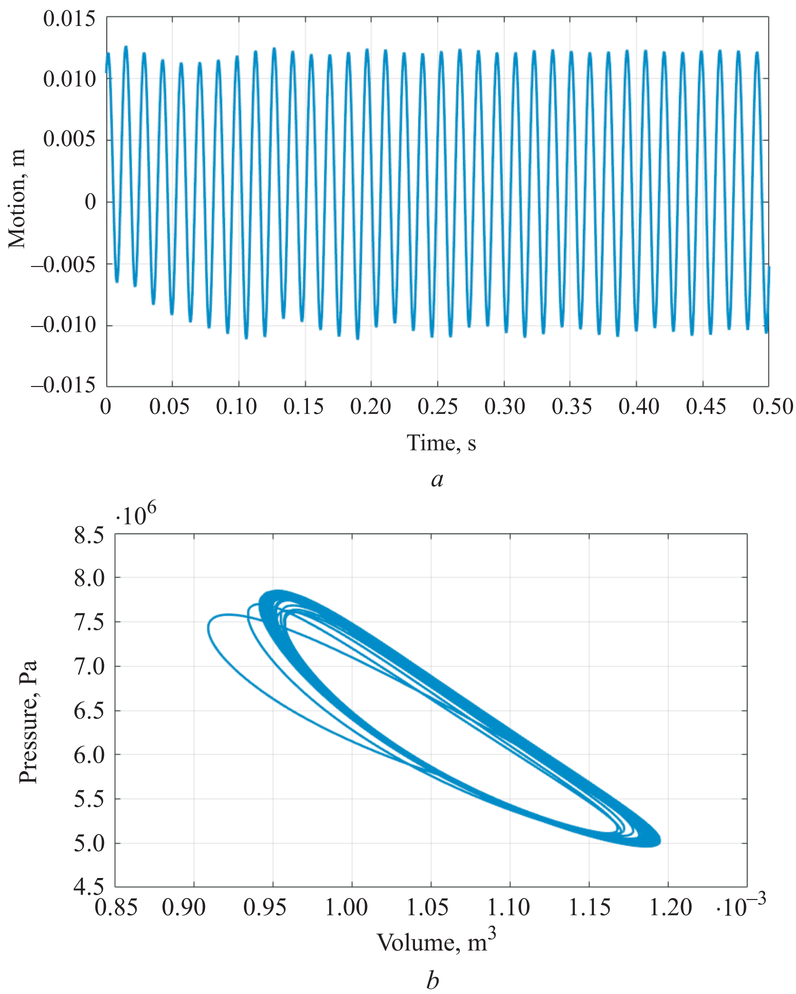

Fig. 6. Calculation results with the selected point parameters:
a is displacer motion; *b* is engine indicator diagram

Discussion. The results obtained demonstrated operability and applicability of the developed pneumatic damper. Comparison of the working process calculation without a damper and with it at the same engine settings showed the damper ability to prevent an increase in the displacer oscillation amplitude.

Conclusion. As a result, thermodynamic mathematical model of a pneumatic damper was elaborated; multicriteria optimization of the damper parameters by the NSGA-II algorithm was carried out; nature of the damper geometrical parameters influence on its dissipative characteristics was revealed; pneumatic damper mathematical model was integrated into the Stirling engine mathematical model; optimal values of the pneumatic damper parameters were selected and the engine working process with selected parameters was calculated; pneumatic damper ability to ensure stable operation of the Stirling engine displacer was demonstrated by calculation.

The authors believe that introduction of a pneumatic damper could increase stability of the free-piston Stirling engine, as well as simplify the task of its design at the early stages of developing a new unit.

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