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LATERAL MOTION OF TOWED UNDERWATER VEHICLE WITHIN THE PROBLEM OF CONTINENTAL SHELF MONITORING

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Abstract

Lateral motion dynamics was studied of a robotic towed underwater system designed to monitor the continental shelf and consisting of a towed vehicle and a tow wireline. In regard to underwater vehicles of the type in question, it is quite correct to represent spatial motion in the form of a super-position consisting of two flat motions, i.e., longitudinal motion in the vertical plane and lateral motion in the horizontal plane. Dynamics of the towed system longitudinal motion within the monitoring problem was considered in a previously published work by the authors. The present work is its natural continuation and development traditionally accepted in the problems of the underwater vehicles spatial motion mechanics. Diagram of the towed vehicle operation and its hydrodynamic characteristics are presented; besides, mathematical model of a wireline and also a model of the wireline-towed vehicle system lateral motion were constructed. Probable steady system motions were analyzed, issues of balancing, as well as those of the towed vehicle dynamic stability when moving at a constant depth were considered. Results of numerical calculations were provided. The results obtained were considered in conjunction with the results of the authors' above mentioned work related to the towed vehicle longitudinal motion and make it possible to select such system parameters that provide the specified character of spatial movements in the process of monitoring the continental shelf taking into consideration the need to perform turns in the horizontal plane at changing directions and to ensure vertical maneuvers when avoiding underwater obstacles

Keywords

Towed underwater vehicle, continental shelf monitoring, lateral motion dynamics, horizontal circulation, motion stability

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Introduction. Problems of analyzing the dynamics of a mechanical system consisting of tow-wireline-towed vehicle motion were considered in works [1–12] and in a number of other papers, which brief review is presented in work [6]. Basically, these works are devoted to studying a system steady longitudinal motion. Statement of the problem raised in this work and devoted to studying the towed vehicle lateral, as well as statement of problem in analyzing its longitudinal motion considered in work [6] during the continental shelf monitoring process, put forward a number of new stringent requirements to a towed system dynamics in comparison with works [1–5] and [7–12]. These requirements include very accurate angular stabilization of the towed vehicle differential longitudinal axis with respect to the given direction, ensuring a wide range of depths in the established horizontal motion and corresponding towered vehicle balancing state under conditions of hydrostatic force excess over gravity, requirement to the wireline length, which should be not less than some limiting length to guarantee an acceptably low level of interference from the towing ship on the towed vehicle acoustic observation system. Finally, there is a requirement to ensure that the towed vehicle could track the sea bottom relief and bypass the underwater obstacles. Paper [6] is devoted to studying these issues in regard to the case of wireline-towed vehicle system longitudinal motion. The present work is a natural continuation and development of paper [6]. It addresses the issues of towed vehicle lateral motion and ensuring the towed vehicle circulation motion in the horizontal plane, which makes it possible to close the system operation scheme. The problem under consideration considers that the wireline ensures balance in the vertical plane, and control in the horizontal plane is carried out by deflecting vertical rudders of the towed vehicle.

Problem statement. As in paper [6], a tow-wireline-towed vehicle mechanical system is considered designed to being used on the continental shelf in order to solve a wide range of problems associated with the sea bottom, such, for example, as searching for various potentially hazardous objects, inspection of underwater engineering structures, studying the underwater relief, creating the depth map, searching for sunken objects and many others. A somewhat more detailed review of possible tasks is provided in papers [1–6].

As in paper [6], it is assumed that one end (running end) of the flexible wireline is fastened to the towed vehicle (TV) with an attachment point, the other (root) end is wound on the winch of the ship winch system installed on board the towing ship. The scheme of tow-wireline-towed vehicle system operation is presented in Fig. 1.

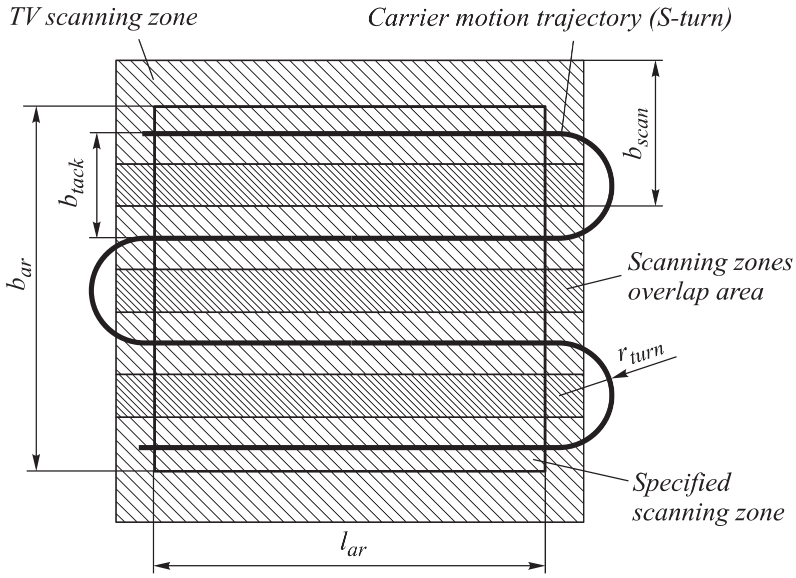

Fig. 1. Tow-wireline-towed vehicle system scheme of operation

The following notations is used here: b_{ar} and l_{ar} are width and length of the specified scanning zone; r_{turn} is the tow trajectory turning radius; b_{tack} is the tow tack width, i.e., the distance between rectilinear sections of its trajectory; b_{scan} is the width of the towed vehicle scanning zone.

When scanning the observation area, the tow moves for a while in a straight line, then turns around and moves in the opposite direction, etc. (see Fig. 1). Moreover, in the problem statement under consideration, the towed vehicle before turning could be moved to the tow using a winch. In other words, scanning of the water basin is carried out by tacks (S-turns). In this case, scanning zones of the towed vehicle should be overlapped with a certain margin for reliable coverage of the entire examined area. The indicated action scheme, in addition to procedures specified in paper [6], includes a turn in horizontal plane carried out after passing through each rectilinear section.

Relative arrangement of the tow-wireline-towed-towed vehicle system elements during a steady-state circulation, as well as the necessary symbolic notations at the wireline balance are shown in Fig. 2.

As in paper [6], it is supposed that the body of a towed vehicle is a feathered body of revolution, on which wings for deepening and steering controls are installed. Geometrical parameters of the towed vehicle, i.e., wingspan, limiting body and empennage linear dimensions and empennage arrangement on the body, are subject to design limitations caused by ensuring the reliable operation of the acoustic system monitoring the underwater space,

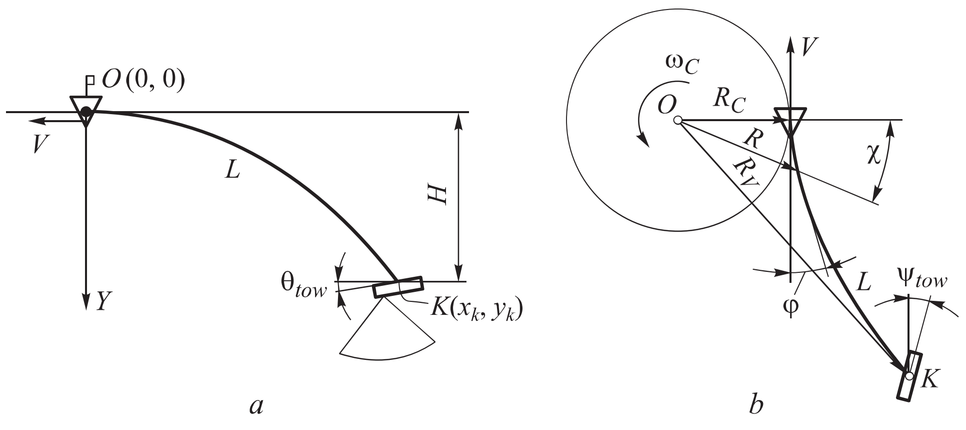

Fig. 2. Scheme of the tow-wireline-towed vehicle system elements arrangement and wireline balance parameters at steady towing in projection on the tow symmetry plane (*a*) and on the horizontal plane (*b*)

as well as by arrangement of the towed vehicle on board the tow, when the towed vehicle is not used for its intended purpose, and the tow itself is operating as the towed vehicle carrier. It is assumed that the towed vehicle motion in the turning section occurs after the towed vehicle is pulled to the tow for a short distance and is in established circulation in the horizontal plane. The water surface is considered flat and undisturbed; no sea currents are present. Wireline (flexible connection) is assumed to be weighty, absolutely flexible, inextensible, having some fixed strength limit determined by the T^* ultimate tensile force. The c_{x0} longitudinal and the c_{x90} transverse hydrodynamic wireline characteristics, as well as the ρ_T wireline material density are considered to be known.

It is required to build algorithms and corresponding relations that make it possible (jointly using the results indicated in paper [6]) to select geometric parameters of the towed vehicle, balancing angle values of the installed wings and controls deflection in such a way that, under conditions of significantly positive towed vehicle buoyancy, $p = A - G > 0,$ steady horizontal motion of the towed vehicle and of the entire towed cable system is ensured in the horizontal plane turning sections, when changing motion direction to the opposite in operating modes that satisfy the restrictions in υ and L parameters.

Model of the towed vehicle hydrodynamic characteristics in spatial motion. Hydrodynamic scheme of the towed vehicle is accepted as it is in paper [6]. In regard to the towed vehicle hydrodynamic characteristics in spatial motion, the following expressions should be accepted:

$$c_{xa}\left(\alpha_k, \beta_k, \delta_\Sigma\right) = c_x\left(\alpha_k, \beta_k\right)\Big|_{\delta_\Sigma = 0} + c_x^\delta\left(\alpha_k, \beta_k\right)\delta_\Sigma;$$

$$c_{ya}\left(\alpha_k, \beta_k, \delta_H\right) = c_y\left(\alpha_k, \beta_k\right)\Big|_{\delta_{RH} = 0} + c_y^\delta\left(\alpha_k, \beta_k\right)\delta_H;$$

$$c_{za}\left(\alpha_k, \beta_k, \delta_V\right) = c_z\left(\alpha_k, \beta_k\right)\Big|_{\delta_{RV} = 0} + c_z^\delta\left(\alpha_k, \beta_k\right)\delta_V;$$

$$m_x\left(\alpha_k, \beta_k, \delta_E\right) = m_x\left(\alpha_k, \beta_k\right)\Big|_{\delta_E = 0} + m_x^{\delta_E}\delta_E;$$

$$m_y\left(\alpha_k, \beta_k, \delta_V\right) = m_y\left(\alpha_k, \beta_k\right)\Big|_{\delta_{RV} = 0} + m_y^\delta\left(\alpha_k, \beta_k\right)\delta_V;$$

$$m_z\left(\alpha_k, \beta_k, \delta_H\right) = m_z(\alpha_k, \beta_k)\Big|_{\delta_{RH} = 0} + m_z^\delta\left(\alpha_k, \beta_k\right)\delta_H,$$

where α_k and β_k are the towered vehicle attack and slip angles; δ_H and δ_V are deflection angles of its horizontal and vertical rudders. Dependences of the spatial velocity hydrodynamic coefficients include: the c_{xa} drag force; the c_{ya} lift force; the c_{za} lateral force; the m_x roll moment; the m_y yaw moment; the m_z different moment of the towed vehicle from α_k and β_k angles obtained experimentally, and are presented (at zero rudder deflection angles) in Fig. 3.

Rotational derivatives and damping coefficients, as in paper [6], are calculated using the method by K.K. Fedyaevsky [13–15].

Wireline model at the tow steady circulation motion. Wireline motion (equilibrium) equations. The general problem of spatial equilibrium in the system under consideration could be divided into two: the problem of the wireline equilibrium and the problem of the towed vehicle equilibrium. Let us consider the first of them.

Since the wireline motion is steady, equations of its motion turn into equilibrium equations during circulation for an elementary wireline section (wireline equilibrium differential equations) in the cylindrical coordinate system [1]:

$$\begin{aligned}
&\frac{dT}{dl} = -c_{x0}\frac{\rho d_T}{2}\omega_C^2 R^2\cos^2\alpha - p\sin\alpha\cos\varphi;\\
&\frac{d\alpha}{dl} = c_{x90}\frac{\rho d_T}{2T}\omega_C^2 R^2\sin^2\alpha - \frac{p}{T}\cos\alpha\cos\varphi - \frac{\cos\alpha\,\sin\varphi}{R};\\
&\frac{d\varphi}{dl} = \frac{1}{\sin\alpha}\left(\frac{p}{T}\sin\varphi - \frac{1}{R}\cos^2\alpha\cos\varphi\right);\\
&\frac{d\chi}{dl} = \frac{\cos\alpha}{R};\;\; \frac{dy}{dl} = \sin\alpha\cos\varphi;\;\; \frac{dR}{dl} = \sin\alpha\sin\varphi.
\end{aligned} \qquad (1)$$

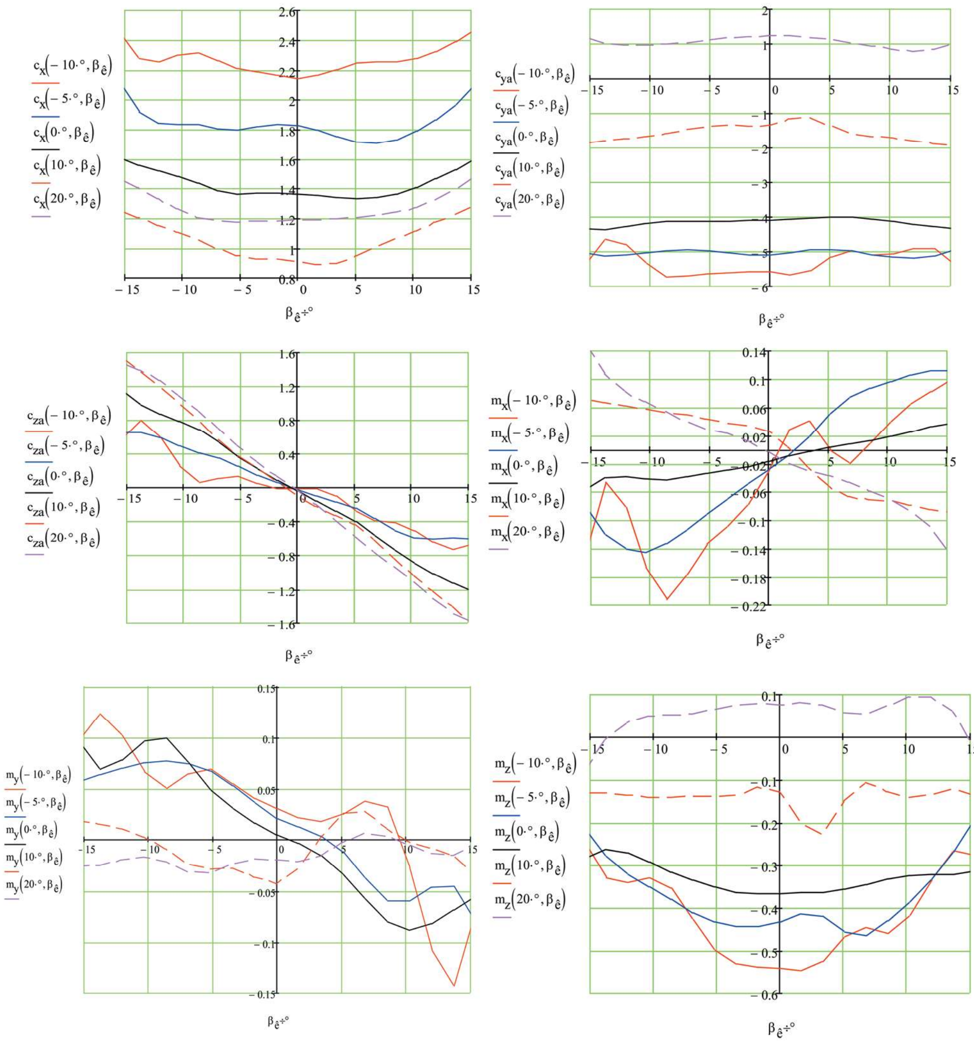

Fig. 3. Dependences of the towed vehicle $c_x, c_y, c_z, m_x, m_y, m_z$ spatial hydrodynamic coefficients on the β_k slip angle at the α_k = –10°, –5°, 0, 10°, 20° angle of attack values (indicated first in parentheses along the ordinate axis) for zero values of the δ_H and δ_V rudder deflection angles

System (1) could be considered in conjunction with the wireline spatial equilibrium equations system in the following form [1]:

$$\frac{dT}{dl} = -r_0 \cos^2 \alpha - p \sin \alpha \cos \varphi; \quad \frac{d\alpha}{dl} = \frac{1}{T}\left(r_{90} \sin^2 \alpha - p \cos \alpha \cos \varphi\right);$$
$$\frac{d\varphi}{dl} = \frac{1}{T \sin \alpha}\left(\varepsilon \sin^3 \alpha \cos \alpha + p \sin \varphi\right); \tag{2}$$
$$\frac{dx}{dl} = \cos \alpha; \quad \frac{dy}{dl} = \sin \alpha \cos \varphi; \quad \frac{dz}{dl} = \sin \alpha \sin \varphi,$$

which in the case of wireline equilibrium in the vertical plane, i.e., for $\varphi \equiv 0$, takes the following form:

$$\frac{dT}{dl} = -r_0 \cos^2 \alpha - p \sin \alpha; \quad \frac{d\alpha}{dl} = \frac{1}{T}\left(r_{90} \sin^2 \alpha - p \cos \alpha\right);$$
$$\frac{dx}{dl} = \cos \alpha; \quad \frac{dy}{dl} = \sin \alpha. \qquad (3)$$

In system (2), ε is the empirical coefficient characterizing the lateral flow around the wireline, and especially in case of a smooth wireline $\varepsilon = 0$ [3].

The following notations are used in the wireline system equations (1)–(3), see Fig. 2: T is the local tension force; α is the local towing angle; φ is the local angle of lateral inclination; χ is the local steering angle of the flow incident on the wireline during circulation; R is the distance from the current wireline point to the tow axis of circulation.

The r_{90}, r_0, p parameters (respectively, the wireline resistance force per unit length at 90° and 0 angles of attack and the wireline mass per unit length in water) are equal to:

$$r_{90} = c_{x90} \frac{\rho V^2}{2} d_T; \quad r_0 = c_{x0} \frac{\rho V^2}{2} d_T; \quad p = \left(\rho_T - \rho\right) \frac{\pi d_T{}^2}{4} g.$$

Signs of the terms in equations (1)–(3) imply a countdown from the root end (connected to the tow) of the wireline.

By attaching boundary conditions (see below) to the system (1) for each of the $T, \alpha, \varphi, \chi, y, R$ wireline equilibrium parameters, a form of its equilibrium could be obtained during circulation for the specified L wireline length, V_C linear and ω_C angular circulation velocities, as well as for the δ_V, δ_H rudder deflection angles.

Towered vehicle spatial motion equations. Let us write down dynamic equations of the towed vehicle spatial motion in the form of two vector equations [14]:

$$\frac{\tilde{d}\vec{Q}}{dt} + \vec{\omega} \times \vec{Q} = \vec{F}_E; \qquad (4)$$

$$\frac{\tilde{d}\vec{K}}{dt} + \vec{\omega} \times \vec{K} + \vec{U} \times \vec{Q} = \vec{M}_E, \qquad (5)$$

where $\vec{Q}$ is the momentum resultant vector; $\vec{K}$ is the towed vehicle momentum resultant moment relative to the beginning of the O_1xyz bound coordinate system; $\tilde{d}\vec{Q}/dt$ and $\tilde{d}\vec{K}/dt$ are local derivatives of the $\vec{Q}$ and $\vec{K}$

vectors in the $Oxyz$ bound coordinate system; $\vec{U}$ is the pole velocity vector of the bound coordinate system relative to the Earth coordinate system; $\vec{\omega}$ is the angular velocity vector of bound axes relative to the Earth axes; $\vec{F}_E$ and $\vec{M}_E$ are the resultant vector and the resultant moment of external forces applied to the towed vehicle.

The $\vec{Q}$ and $\vec{K}$ vectors are determined by the following expressions:

$$\vec{Q} = m\left(\vec{U} + \vec{\omega} \times \vec{r}_c\right); \tag{6}$$

$$\vec{K} = J\vec{\omega} + m\left(\vec{r}_c \times \vec{U}\right), \tag{7}$$

where $\vec{r}_c$ is the radius vector of the towed vehicle center of mass in the bound axes, $\vec{r}_c = \vec{i}x_c + \vec{j}y_c + \vec{k}z_c$; J is the towed vehicle inertia tensor relative to the axes passing through its center of mass and parallel to the bound axes.

System of the towed vehicle motion should be closed in numerical integration by the known kinematic relations [14], which are not presented here for their obviousness, and should be written down in projections on the axis of the bound coordinate system.

Towed vehicle steady circulation. Let us accept the following assumptions for the towed vehicle steady horizontal circulation [13–15]:

$$\omega_x = \omega_z = 0, \ \ \omega_y \approx \omega_C = \text{const}, \ \ \gamma = 0.$$

Moreover, from the last equality and taking into account the conditions of motion at a constant depth:

$$\operatorname{tg}\vartheta \cos\alpha_k - \cos\gamma \sin\alpha_k - \sin\gamma \ \operatorname{tg}\beta = 0$$

it follows that $\alpha_k = \vartheta$.

Then, equilibrium equations of the towed vehicle in circulation would take the following forms:

$$\begin{gathered}
X_t + C_{xa}\left(V_C, \alpha_k, \beta_k, \delta_V, \delta_H\right)\frac{\rho V_C^2}{2} S_m - (G - A)\sin\vartheta - \\
- \left(m + \lambda_{33}\right)\omega_y V_z - \left(\lambda_{35} - m x_c\right)\omega_y^2 = 0; \\
C_{ya}\left(\alpha_k, \delta_H\right)\frac{\rho V_C^2}{2} S_m + Y_t - (G - A)\cos\vartheta = 0; \\
C_{za}\left(V_C, \beta_k, \delta_V, \omega_y\right)\frac{\rho V_C^2}{2} S_m + Z_t + \left(m + \lambda_{11}\right)\omega_y V_x = 0; \\
m_y\left(V_C, \beta_k, \delta_V, \omega_y\right)\frac{\rho V_C^2}{2} S_m L_k - \left(m x_c - \lambda_{35}\right)\omega_y V_x = 0;
\end{gathered} \tag{8}$$

$$m_z\left(\alpha_k, \delta_H\right)\frac{\rho V_C^2}{2} S_m L_k + \left[\left(x_{CV} - x_K\right)A - \left(x_C - x_K\right)G\right]\cos\vartheta - \\ - \left[\left(y_{CV} - y_K\right)A - \left(y_C - y_K\right)G\right]\sin\vartheta = 0, \quad (8)$$

where the attached masses and the attached static moments could be calculated on the basis of paper [16].

Moments in the last two equations of system (8) are registered relative to the K point (see Fig. 1), i.e., in such a way as not to be dependent on the X_t, Y_t, Z_t towing force values taken in the projections on the towed vehicle bound axes.

Taking into consideration the last remark, system (8) is being divided: the last two equations form a closed system for the given V_C, δ_V, δ_H and ω_C, and could be solved independently of the first three, which provides the α_k, β_k desired values.

The first three equations (8), taking into account dependencies for the projections of the towed vehicle generalized velocities on the bound axes

$$V_x = V_C \cos\alpha_k \cos\beta_k \text{ and } V_z = V_C \sin\beta_k,$$

in their decision will give the X_t, Y_t, Z_t towing forces values equal to:

$$\begin{aligned} X_t &= T_\xi \cos\alpha_k \cos\beta_k + T_\eta \sin\alpha_k - T_\zeta \cos\alpha_k \sin\beta_k; \\ Y_t &= -T_\xi \sin\alpha_k \cos\beta_k + T_\eta \cos\alpha_k + T_\zeta \sin\alpha_k \sin\beta_k; \\ Z_t &= T_\xi \sin\beta_k + T_\zeta \cos\beta_k, \end{aligned} \quad (9)$$

where T_ξ, T_η, T_ζ is the towing force essence in projections on the axis of the $O\xi\eta\zeta$ coordinate system connected to the tow. After solving system (9) in regard to the T_ξ, T_η, T_ζ values, it is possible to calculate the T_K, α_K, φ_K values of the T, α, φ wireline equilibrium parameters at its K running end, which are required for solving system (1):

$$T_K = \sqrt{T_\xi^2 + T_\eta^2 + T_\zeta^2}; \ \alpha_K = \operatorname{arctg}\frac{\sqrt{T_\eta^2 + T_\zeta^2}}{T_\xi}; \ \varphi_K = \operatorname{arctg}\frac{T_\zeta}{T_\eta}.$$

However, as was indicated, the system (8), which provides values of the X_t, Y_t, Z_t forces in its solution, and thereby the T_ξ, T_η, T_ζ values, should be solved not only when the δ_V, δ_H rudders deviation angles are set, but also at the V_C towed vehicle circulation velocity equal to $V_C = \omega_C R_C$. In turn, the V_C and R_C parameters themselves are, as a rule, not known in advance (except for the case when the R_C^* required radius of the towed vehicle circulation is specified), and only the R_{tow} and $V_{tow} = \omega_C R_{tow}$ corresponding parameters in regard to the tow are defined. In the case when the R_{tow} tow circulation radius is specified, the problem of finding the wireline equilibrium form turns out to be a boundary problem and requires an iterative procedure for jointly solving the towed vehicle equilibrium (8) and the wireline (1) equation system at horizontal circulation of the mechanical system. Moreover, as boundary conditions for the χ and y parameters of system (1), the following could be set:

$$\chi\big|_K = 0 \text{ and } y\big|_K = 0.$$

Fig. 4 presents comparative graphs illustrating some results of calculating the wireline equilibrium parameters for both options in setting the system (1) boundary conditions with respect to the R parameter. In this case, the lower index A in the notation of parameters in the graphs corresponds to the R_{tow} setting and index B — to the R_C^* setting. Let us also note that along abscissa in all the graphs the l curvilinear coordinate values are marked, i.e., the wireline current length is counted from its K running end. The α, φ, χ wireline angular parameters are given in degrees.

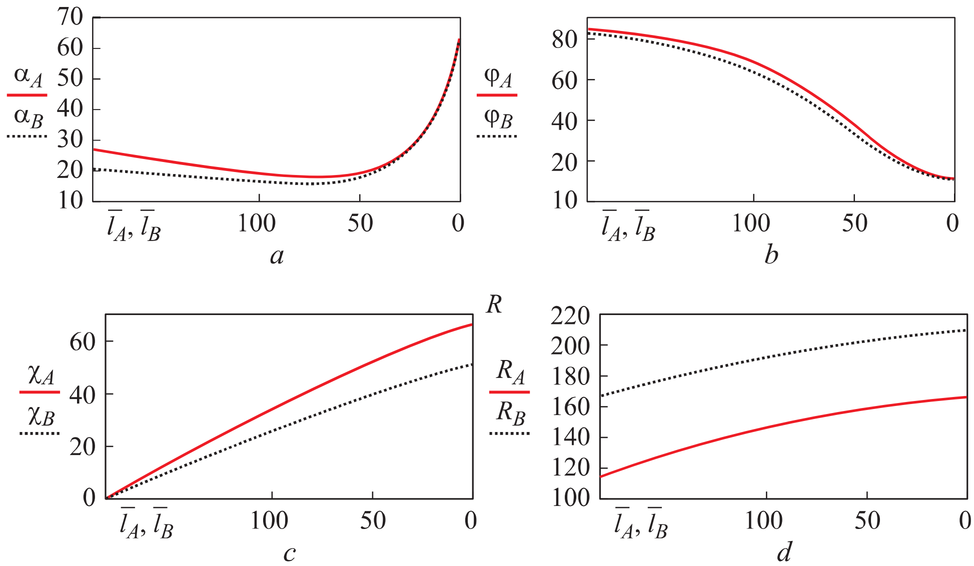

Fig. 4. Alterations in the wireline $\alpha(a)$, $\varphi(b)$, $\chi(c)$, $\bar{R}(d)$ equilibrium parameters during the tow-wireline-towed vehicle system steady circulation characterized by the following values: the $\bar{L} = 180$ wireline dimensionless length, the $\bar{R}_C = \bar{R}_{tv} = 170$ initial dimensionless radii, the $\omega_C = 0{,}7$ °/s angular circulation velocity, $V_C = 6$ m/s linear circulation velocity and the $c_{xa} = 2{,}5$; $c_{ya} = 5$; $c_{za} = -1$ hydrodynamic coefficient values

Lateral stability of TV steady motions. When studying stability of steady motion, where:

$$V_0 = |V| = \text{const},\ \omega_y = \omega_{y0} = \omega_C = \text{const},\ \beta = \beta_{k0} = \text{const},\ \psi = \psi_0,$$

the system of first approximation in regard to equations of the lateral perturbed controlled motion in the horizontal plane at zero roll is written down in the following form [13]:

$$\left(b_{11}p_t + b_{12}\right)\Delta V + b_{13}p_t\Delta\psi + \left(b_{15}p_t + b_{16}\right)\Delta\beta_k = 0;$$

$$\left(b_{21}p_t + b_{22}\right)\Delta V + \left(b'_{23}p_t + b'_{24}\right)\Delta\omega_y + \left(b_{26}p_t + b_{27}\right)\Delta\beta_k = b_{28}\Delta\delta_V;$$

$$\left(b_{31}p_t + b_{32}\right)\Delta V + \left(b'_{33}p_t + b'_{34}\right)\Delta\omega_y + \left(b_{36}p_t + b_{37}\right)\Delta\beta_k = b_{38}\Delta\delta_V,$$

where $b'_{ik} = \dfrac{b_{ik}}{\cos\left(a_{k0}\right)}$, $p_t = \dfrac{d}{dt}$ is the time differentiation operator.

Under the assumption that $\Delta V \equiv 0$, the system of lateral perturbed motion equations takes the following form:

$$\left(b'_{23}p_t + b'_{24}\right)\Delta\omega_y + \left(b_{26}p_t + b_{27}\right)\Delta\beta_k = 0;$$

$$\left(b'_{33}p_t + b'_{34}\right)\Delta\omega_y + \left(b_{36}p_t + b_{37}\right)\Delta\beta_k = 0.$$

In regard to the controlled lateral motion under perfect control, and when the control law could be written down as:

$$\Delta\delta_V = k_1\Delta\psi + k_2 p_t\Delta\psi,$$

the following equations of the perturbed motion first approximation are obtained:

$$\left[b_{23}p_t^2 + \left(b_{24} - k_2b_{22}\right)p_t - k_1b_{28}\right]\Delta\psi + \left(b_{26}p_t + b_{27}\right)\Delta\beta_k = 0;$$

$$\left[b_{33}p_t^2 + \left(b_{34} - k_2b_{32}\right)p_t - k_1b_{38}\right]\Delta\psi + \left(b_{36}p_t + b_{37}\right)\Delta\beta_k = 0,$$

where

$$b_{22} = -\left(m + \lambda_{11}\right)\omega_{y0}\cos\alpha_{k0}\cos\beta_{k0} -$$
$$-\rho S V_0\left(c_z^{\beta}\beta_{k0} + c'_3\beta_{k0}^3 + c_z^{\delta}\delta_B\right) - c_z^{\overline{\omega}_y}\frac{\rho S L}{2}\omega_{y0};$$

$$b_{23} = -\left(\lambda_{26} + mx_c\right)\cos\alpha_{k0};$$

$$b_{24} = \cos\alpha_{k0}\left[-\left(m + \lambda_{11}\right)\cos\alpha_{k0}\cos\beta_{k0} - c_z^{\overline{\omega}_y}\frac{\rho S L}{2}\right]V_0;$$

$$b_{26}=(m+\lambda_{11})V_0\cos\beta_{k0};$$

$$b_{27}=\left[(m+\lambda_{11})\omega_{y0}\cos\alpha_{k0}\sin\beta_{k0}-\frac{\rho SV_0^2}{2}\right]\left(c_z^{\beta}\beta_{k0}+3\beta_{k0}^2c_3'\right);$$

$$b_{28}=c_z^{\delta}\frac{\rho S}{2}V_0^2;$$

$$b_{32}=(m+\lambda_{11})\omega_{y0}\cos\alpha_{k0}\cos\beta_{k0}-$$

$$-\left(m_y^{\beta}\beta_{k0}+m_3'\beta_{k0}^3+m_y^{\delta}\delta_{VB}\right)\rho SLV_0-m_y^{\bar{\omega}_y}\frac{\rho SL}{2}\omega_{y0};$$

$$b_{33}=(I_y+\lambda_{55})\cos\alpha_{k0};$$

$$b_{34}=\left[(\lambda_{26}+mx_c)V_0\cos\alpha_{k0}\cos\beta_{k0}-m_y^{\bar{\omega}_y}\frac{\rho SL}{2}\right]\cos\alpha_{k0};$$

$$b_{36}=-(\lambda_{26}+mx_c)V_0\cos\beta_{k0};$$

$$b_{37}=-(\lambda_{26}+mx_c)\omega_{y0}V_0\cos\alpha_{k0}\sin\beta_{k0}-\frac{\rho SL}{2}V_0^2\left(m_y^{\beta}+3m_3'\beta_{k0}^2\right);$$

$$b_{38}=m_y^{\delta}\frac{\rho SL}{2}V_0^2.$$

Introducing new notations:

$$a=b_{23},\ b=(b_{24}-k_2b_{22}),\ c=k_1b_{28},\ d=b_{26}p+b_{27},$$

$$a_1=b_{33},\ b_1=(b_{34}-k_2b_{32}),\ c_1=k_1b_{38},\ d_1=b_{36}p+b_{37},$$

we obtain

$$\left[ap_t^2+bp_t-c\right]\Delta\psi+d\Delta\beta_k=0;$$

$$\left[a_1p_t^2+b_1p_t-c_1\right]\Delta\psi+d_1\Delta\beta_k=0.$$

It follows that stability conditions have the following form:

$$D_k>0,\ k=\overline{0,3};\quad D_1D_2-D_0D_3>0,$$

where

$$D_3=b_{23}b_{36}-b_{33}b_{26};$$

$$D_2=b_{23}b_{37}+(b_{24}-k_2b_{28})b_{36}-b_{33}b_{27}-(b_{34}-k_2b_{38})b_{26};$$

$$D_1=(b_{24}-k_2b_{28})b_{37}-k_1b_{28}b_{36}-(b_{34}-k_2b_{38})b_{27}-k_1b_{38}b_{36};$$

$$D_0=-k_1b_{26}b_{36}+k_1b_{38}b_{27}.$$

Conclusions. A mathematical model was constructed in regard to steady horizontal circulation motion of an underwater towed vehicle within the problem of monitoring the continental shelf, as well as the algorithm for such motion determination. Possibility of realizing these movements was demonstrated taking into consideration fulfillment of the balancing conditions. The problem was solved in constructing the lateral dynamic stability conditions for the considered horizontal steady motion of a towed system designed to monitor the near-bottom shelf area taking into account both strict restrictions on the range of depths of the towed vehicle motion and on the trim angles, and significant positive buoyancy of the towed vehicle. Together with the results of paper [6], the constructed algorithms make it possible to solve the problems of spatial towing of a towed vehicle of the type under consideration.

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REFERENCES

[1] Vinogradov N.I., Gutman M.L., Lev I.G., et al. Privyaznye podvodnye sistemy. Prikladnye zadachi statiki i dinamiki [Tethered submersibles. Applied problems of statics and dynamics]. St. Petersburg, Izd-vo SPbGU Publ., 2000.

[2] Vinogradov N.I., Kreyndel' S.A., Lev I.G., et al. Privyaznye podvodnye sistemy. Aerogidrodinamicheskie kharakteristiki pri ustanovivshemsya dvizhenii [Tethered submersibles. Aerohydrodynamic characteristics at stationary motion]. St. Petersburg, FGUP "TsNII "Gidropribor" Publ., 2005.

[3] Egorov V.I. Podvodnye buksiruemye sistemy [Towed underwater system]. Leningrad, Sudostroenie Publ., 1981.

[4] Kuvshinov G.E. Upravlenie glubinoy pogruzheniya buksiruemykh ob"ektov [Control on immersion depth of towed objects]. Vladivostok, Izd-vo Vladivostokskogo un-ta Publ., 1987.

[5] Poddubnyy V.I., Shamarin Yu.E., Chernenko D.A., et al. Dinamika podvodnykh buksiruemykh system [Dynamics of towed underwater systems]. St. Petersburg, Sudostroenie Publ., 1995.

[6] Grumondz V.T., Pilgunov R.V., Vinogradov M.V. Longitudinal dynamics of underwater towed equipment in the problem of monitoring the defined region of the continental shelf. *Vestn. Mosk. Gos. Tekh. Univ. im. N.E. Baumana, Mashinostr.* [Herald of the Bauman Moscow State Tech. Univ., Mechan. Eng.], 2017, no. 6, pp. 19–34 (in Russ.).
DOI: 10.18698/0236-3941-2017-6-19-34

[7] Merkin D.R. Vvedenie v mekhaniku gibkoy niti [Introduction into elastic yarn mechanics]. Moscow, Nauka Publ., 1980.

[8] Devnin S.I. Aerogidromekhanika plokhoobtekaemykh konstruktsiy [Aerohydromechanics of high-drag constructions]. Leningrad, Sudostroenie Publ., 1983.

[9] Hegenuier G., Nair S. A nonlinear dynamic theory for heterogenius, anisotropic, elastic rods. *AIAA Journal*, 1977, vol. 15, no. 1, pp. 8–15. DOI: 10.2514/3.7296

[10] Pole B., Soler A. Cable dynamics and optimum towing strategies for submersibles. *MTS Journal*, 1972, vol. 6, no. 2, pp. 34–42.

[11] Henderson J.F. Some towing problems with faired cables. *Ocean Eng.*, 1978, vol. 5, no. 2, pp. 105–125. DOI: 10.1016/0029-8018(78)90064-1

[12] Nair S., Hegenuier G. Stability of faired underwater towing cables. *J. Hydronautics*, 1979, vol. 13, no. 1, pp. 20–27.

[13] Pantov E.N., Makhin P.N., Sheremetov B.B. Osnovy teorii dvizheniya podvodnykh apparatov [Theory fundamentals of submersible vessel mootion]. Leningrad, Sudostroenie Publ., 1973.

[14] Grumondz V.T., Polovinkin V.V., Yakovlev G.A. Teoriya dvizheniya dvusrednykh apparatov. Matematicheskie modeli i metody issledovaniya [Theory of two-medium apparatus motion. Mathematical models and research methods]. Moscow, Vuzovskaya kniga Publ., 2012.

[15] Grumondz V.T., Yakovlev G.A. Algoritmy aerogidroballisticheskogo proektirovaniya [Aerohydrodynamical engineering algorithms]. Moscow, Izd-vo MAI Publ., 1994.

[16] Korotkin A.I. Prisoedinennye massy sudostroitel'nykh konstruktsiy [Added mass of shipbuilding constructions]. St. Petersburg, MorVest Publ., 2007.

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